WORKING paper

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EUROSYSTÈME

The FR-BDF Model and an Assessment of Monetary Policy Transmission in France

Matthieu Lemoine[1,2], Harri Turunen[1], Mohammed Chahad[1], Antoine Lepetit[1], Anastasia Zhutova[1], Pierre Aldama[1], Pierrick Clerc[1] & Jean-Pierre Laffargue[3]

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Abstract

This paper presents the new model for France of the Banque de France (FR-BDF), as well as its key implications for the analysis of monetary policy transmission in France. Relative to our former model, this new semi-structural model has been improved along three dimensions: financial channels are richer, expectations now have an explicit role and simulations now converge toward a balanced growth path. We follow the approach of the FRB/US model, where agents can form their expectations in two different ways, VAR-based or model-consistent, and where non-financial behavior react with polynomial adjustment costs. For standard monetary policy shocks, FR-BDF shows a stronger sensitivity than our former model, due to the widespread influence of expectations. Then, we show that, under model-consistent expectations, FR-BDF does not suffer from the forward guidance puzzle. Finally, Eurosystem asset purchase programmes have notable effects in FR-BDF, with a stronger transmission through exchange rates than term premia.

Keywords: semi-structural modeling, expectations, monetary policy, forward guidance

JEL classification: C54, E37

[1] Banque de France
[2] Corresponding author, matthieu.lemoine@banque-france.fr
[3] University of Paris 1

Authors are listed according to the duration of their participation in the project, but authors who participated for similar durations are listed alphabetically (M. Chahad, A. Lepetit and A. Zhutova). We would particularly like to thank Jean-François Ouvrard for his continuous-time feedback on this paper and on the whole modeling project behind it. We also have special thanks for Stéphane Dees, Yannick Kalantzis, Pierre Sicsic and Camille Thubin for their careful reread and detailed comments. We also thank Werner Roeger for useful discussion and comments. This paper is the outcome of a modeling project of the Banque de France with a well-defined governance and many contributors, which we expand on in the special acknowledgment.

NON-TECHNICAL SUMMARY

The model for France of the Banque de France (FR-BDF) is the new semi-structural replacement of the older model Mascotte. FR-BDF is a large-scale model for France, which contains detailed behavioral equations as well as a detailed accounting framework. It is used both for medium-run projection exercises and for policy analysis. The French economy is modeled as a small-open economy under fixed exchange rates with an exogenous interest rate due to the constraints of the Eurosystem projection framework. The overall model structure is strongly inspired by the US model of the Federal Reserve Board (FRB/US). This model combines economically meaningful expectations, a good empirical fit due to the Polynomial Adjustment Costs (PAC) framework and ease of estimation due to the separability of model blocks.

Three key features should be emphasized among the improvements of FR-BDF with respect to Mascotte. *First*, FR-BDF has richer financial channels than Mascotte. It has a large set of interest rates, an endogenous term structure and endogenous nominal exchange rates. *Second*, expectations play an explicit role, both for financial and non-financial variables, as expectations are a major transmission channel of monetary policy shocks. Expectations can be modeled based on a vector autoregressive model (VAR) which summarizes the state of the economy. We can also allow expectations to be consistent with the model and forward-looking, which we label Model-Consistent Expectations (MCE). *Third*, FR-BDF has a well-defined supply block as well as a balanced growth path toward which it converges smoothly and endogenously in unconditional simulations.

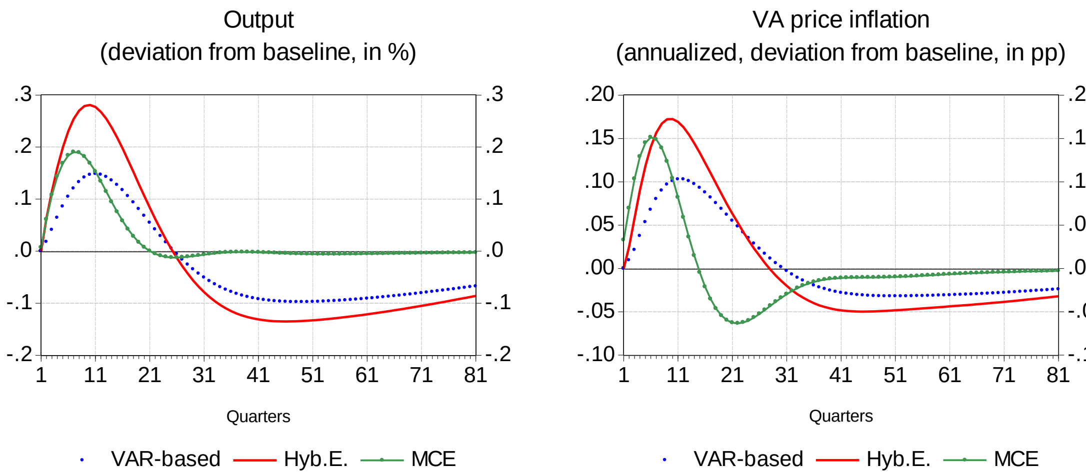
Concerning short run dynamics of FR-BDF under VAR-based expectations, one striking feature of our impulse responses to positive and temporary demand shocks is related to the absence of any feedback from monetary policy. The expansionary effect of these shocks deteriorates the competitiveness, weights on the output gap, leading later to disinflation and bringing the real exchange rate back to its baseline level. Impulse responses to supply shocks illustrate the important role played by the supply block of this model at horizons that could matter for medium-run forecasts. For example, after a shock that gradually raises labor efficiency by 1%, real output increases by 0.6 percentage points over a 4-year horizon.

In a last section, we analyze monetary policy transmission in France with three experiments through the lens of FR-BDF. First, we assess the impact of a standard monetary policy shock. FR-BDF shows a stronger response than our former model and this can be related to the influence of the short term rate through expectations. There are some differences in outcomes that depend on how expectations are modelled (see figure): when non-financial variables are modeled in a forward-looking manner (full MCE case), this creates a dampening effect in comparison to the case where they would be backward looking (hybrid case).

Second, we simulate FR-BDF with model-consistent expectations in order to assess the impact in France of forward guidance in the form of an announced cut in short-term interest rate for a varying amount of quarters. It appears that FR-BDF does not suffer from the forward guidance puzzle, i.e. the impact of forward guidance increases linearly and not exponentially with the length of the shock, thanks to the small sensitivity of consumption to interest rates and to discounting schemes used in consumption and term structure equations.

The final experiment we consider deals with how Eurosystem Asset Purchase Programmes (APP) have affected the French economy between 2015 and 2018. We only take into account at this stage direct effects on the French economy and do not those coming from the rest of the euro area. Our main results show that APP had notable real and nominal effects on the French economy, with exchange rates as a stronger channel of transmission than term premia.

Monetary policy responses under different types of expectations



Note: responses for VAR-based, hybrid (Hyb.E) and model-consistent expectations (MCE). Hybrid expectations mix VAR-based expectations for non-financial variables and MCE for financial ones.

Le modèle FR-BDF et une évaluation des effets de la politique monétaire en France

RÉSUMÉ

Cet article présente le nouveau modèle pour la France de la Banque de France (FR-BDF), ainsi que ses implications pour l'analyse de la transmission de la politique monétaire en France. Par rapport à notre modèle précédent, le nouveau modèle semi-structurel a été amélioré dans trois dimensions: les canaux financiers sont plus riches, les anticipations ont à présent un rôle explicite et les simulations convergent à présent vers un chemin de croissance équilibrée. Nous suivons l'approche du modèle FRB/US, dans laquelle les agents forment leurs anticipations de deux façons, à l'aide d'un modèle VAR ou du modèle lui-même, et où les comportements non financiers réagissent avec des coûts d'ajustement polynomiaux. Pour des chocs de politique monétaire standard, FR-BDF témoigne d'une sensibilité plus forte que notre modèle antérieur, en raison de la large influence des anticipations. Nous montrons aussi que, avec des anticipations cohérentes avec le modèle, FR-BDF ne souffre pas du *forward guidance puzzle*. Enfin, les programmes d'achats d'actifs de l'Eurosystème auraient eu selon FR-BDF des effets notables, avec une transmission passant plus fortement par les taux de change que par les primes de terme.

Mots-clés : modélisation semi-structurelle, anticipations, politique monétaire, guidage prospectif

Special acknowledgment

This paper is the outcome of a modeling project with a well-defined governance and many contributors. As it owes much to many persons within the Banque de France, we would like to provide here a detailed description of these contributions.

Regarding the governance, first, all choices were continuously validated by M. Lemoine (head of the project) and cross-validated by Jean-François Ouvrard (head of the forecasting unit) as the first user of the model. At the very beginning of the project, its design also benefited from inputs from H. Le Bihan (the former head of the forecasting unit). Second, methods and results were also regularly approved by the former director of macroeconomic analysis and forecasting, P. Sicsic, the current director, G. Levy-Rueff, as well as his deputy, Y. Kalantzis. Third, major choices have also been discussed and validated in an Oversight Committee, headed by O. Garnier, DG Statistics, Economics and International, and previously by the former DG, M.-O. Strauss-Kahn.

Regarding the contributors to the project, the core contributors were the authors of this document, i.e. the members of the modeling team in charge of developing the model: the current members (P. Aldama, H. Turunen and A. Zhutova, headed by M. Lemoine), with the consultancy of J.-P. Laffargue, as well as earlier members of this team (M. Chahad, P. Clerc and A. Lepetit). From the start of the project, this modeling team has been supported by the excellent assistance of L. Giuliani for the development of databases and the implementation of the accounting framework.

The project also benefited from the expertise of our colleagues on specific topics: C. Thubin on business investment; G. Gaulier and B. Meunier on external trade; H. Camatte on consumption; M. Cochard on labor market; E. Monnet and V. Faubert on household investment; A. Devulder, V. Chouard and M. Lequien on the supply side; C. Malgouyres on wages; J. Matheron, H. Le Bihan and Sarah Mouabbi on the financial block; M. Aouriri and Thao Vu on public finances; M. Juillard on simulations under model-consistent expectations; H. Le Damany on equations of demand deflators; Magali Marx and Paul Vertier on interactions with HICP forecasts; C. Wolf, S. Périllaud and E. Lor-Lhommet on the insertion of the model in the forecasting infrastructure.

Finally, we would like to thank economists from other institutions for useful exchanges: Matteo Ciccarelli, Nikola Bokan, Kai Christoffel and Srecko Zimic for the regular exchanges we have had during our parallel ECB and Banque de France modeling projects; Flint Brayton, Hess Chung and Jean-Philippe Laforte for their explanations about FRB/US and their feedback on FR-BDF; José Dorich and his colleagues for their presentations of Bank of Canada models at the very beginning of the project.

Contents

1 Introduction

The model for France of the Banque De France (FR-BDF) is the new semi-structural replacement of the older medium term macro model Mascotte (Baghli et al., 2004). Due to challenges posed by the modern economy, certain features of Mascotte had become unsatisfactory: the sensitivity of this model to monetary and financial shocks is very weak, while such shocks are seen as major drivers of the business cycle since the Great Recession. Moreover, the traditional macro-econometric approach of Mascotte misses important transmission channels of monetary policy related to expectations. As we believe that these missing channels are too important for minor revisions to be satisfactory, the modeling framework should be completely overhauled. This is the starting point of the FR-BDF project.

Before explaining our new approach, we can explain what is the purpose for Banque de France of having a large-scale model for France, which contains detailed behavioral equations as well as a detailed accounting framework (around 400 equations in total). First, for internal purposes, we need a detailed analysis of transmission channels, both for forecasting and counterfactual exercises. For example, in recent forecasts, it was useful to provide a detailed decomposition of the effect of fiscal packages related to the yellow vests crisis on households' real disposable income. Second, this level of detail is also important for our participation in the Eurosystem's bi-annual broad macroeconomic projections exercises (BMPE), in which all national central banks of the Eurosystem build the scenario of their own country for a standardized set of variables (around 90 quarterly variables) using a common set of assumptions (Brent price, exchange rates, world demand, etc.).

Participating in the Eurosystem projections also has some other consequences for our modeling choices. Although France accounts for roughly 20% of the euro area GDP, a conscious choice was made to keep the small open economy setup, where French developments have no influence on euro area monetary policy and the wider international economy, as was the case in Mascotte. The main reason for this choice is that FR-BDF will always be used in interaction with models from other national central banks and with technical Eurosystem assumptions, when conducting Eurosystem operational forecasts. Still, we allow the possibility of endogenizing some parts of the external environment for some counterfactual experiments, e.g. the response of exchange rates to monetary policy shocks.

Hence, the modeling project started with several goals in mind. The two most important targets to be met were: (i) to generate reasonable and detailed conditional forecasts and (ii) to provide detailed counterfactual scenarios. In particular, FR-BDF should be able to model the consequences of monetary policy and financial shocks accurately, and should thus have an explicit role for expectations and a richer financial block. Another desired feature was to have a smoother convergence to a well-defined balanced-growth path (BGP) in unconditional simulations, in order to clarify the distinction between expert judgment and model output. Indeed, judgment should be used more for including external information than for ensuring such a convergence.

The overall model structure is strongly inspired by the US model of the Federal Reserve Board (FRB/US). This type of model combines economically meaningful expectations, a good empirical fit due to the Polynomial Adjustment Costs (PAC) framework, ease of estimation due to the separability of model blocks and the possibility of having a well-defined long-term balanced growth path in general equilibrium. In this approach, we can allow different types of expectations: expectations based on a small Vector Autoregression (VAR) that summarizes the state of the economy, model-consistent expectations (MCE) and hybrid expectations that would be based on either type of expectation depending on agent types. Still, the main difference between the French and the US economies that has to be reflected in FR-BDF is clearly the absence of independent monetary policy in the case of France.

In order to achieve the goals described above, we also paid particular attention to the financial block of the model. First, the term structure, as well as exchange rates, are endogenous, in order to properly analyze the transmission of monetary policy shocks. Second, the structure of interest rates is richer which considerably helps to capture the important role of financial shocks.

In comparison with the forecasting models of other major central banks, such as FRB/US, FR-BDF lies halfway between the fully micro-founded DSGEs[1] which put more the emphasis on theoretical consistency and the traditional semi-structural models[2] which put more emphasis on the empirical fit. Compared to other semi-structural models, the FRB/US approach also seeks to achieve a balance between theoretical consistency and data fit, but it requires additional constraints related to the explicit role of expectations. This FRB/US approach has also been adopted in some other major central banks: it replaced the DSGE approach for forecasting exercises at the Bank of Canada; it has also been followed by the ECB for the new revision of its semi-structural models involved in the forecasting process.[3]

Contrary to the Banque de France, other major French institutions have not included explicit expectation channels in the recent versions of their forecasting models, but have focused more on channels that are less important for a central bank than those related to the transmission of monetary policy. For example, the last version of the Insee-Treasury model (Mésange) has been enriched with labor skill segmentation (Bardaji et al., 2017), an important feature for assessing the impact of labor income tax cuts targeted at low-skilled workers. Another example is the last version of the OFCE model (e-mod), which has been enriched with non-linear hysteresis effects in the labor market (Creel et al., 2011) as a way of capturing the increase in fiscal multipliers during recessions.

The model can also be positioned using conceptual frameworks recently presented in the academic literature. Blanchard (2018) seeks to classify modern macroeconomic general equilibrium models into five partially overlapping groups: "core", "foundation", "policy", "toy" and "forecasting". Within this classification, even though it is mostly used to forecast the French economy, FR-BDF is solidly within the "policy" group and, more generally, the model is intended to be able to provide meaningful answers to counterfactual questions on the dynamics of the economy.

In this paper, we present the model and its main properties – long-term convergence and main impulse responses – and we describe the dynamics of the model regarding the transmission of standard and nonstandard monetary shocks, as this type of simulation yielded

[1]For examples, see the COMPASS model of the Bank of England, RAMSES of the Riksbank and AINO of the Bank of Finland.

[2]For examples, see the Makro model of the Bundesbank, BIQM of the Banca d'Italia, MTBE of the Banco de España and DELFI 2.0 of De Nederlandsche Bank.

[3]These models are respectively ECB-BASE for the euro area version and ECB-MCM for the multi-country version. The motive of the ECB for extending their DSGE toolbox and including such semi-structural models is explained in detail in the speech of Constâncio (2017), which can be found here.

particularly unsatisfactory results with our former model. For the analysis of the model's main properties, we rely on VAR-based expectations, i.e. the type of expectations that we use in a forecasting context. Then, for experiments related to the analysis of monetary policy transmission, we also carry out simulations under model-consistent expectations, as these expectations might be necessary for capturing the short run impact of announcements related to the persistence of shocks.

As regards the model's main properties, we start by checking the convergence of unconditional simulations of FR-BDF toward a balanced growth path (BGP). As shown by our simulations, the output and inflation gaps converge toward zero over a period of around forty years. In the short run, the dynamics of FR-BDF can deviate from long run targets due to nominal and real rigidities modeled with PAC. As a result, convergence toward the BGP is achieved once both nominal and real rigidities have vanished. To understand such a slow convergence, we should keep in mind two key features of the model: in the absence of an independent monetary policy, these gaps are only closed by very slow price-competitiveness mechanisms; the dynamics of these variables are also influenced by stock variables, i.e. capital services and net financial assets, which have very inertial dynamics.

We then turn to the short run dynamic properties of FR-BDF and present four demand shocks (short rate, term premium, foreign demand and public consumption) and three supply shocks (cost-push on VA price, oil price and labor efficiency). Compared to the impulse responses of e.g. FRB/US, one striking feature of our impulse responses to positive and temporary demand shocks is related to the absence of any feedback from monetary policy and nominal exchange rates. The expansionary effect of these shocks on output and inflation deteriorates the competitiveness of exports. Net exports and output gap turn negative, leading to disinflation and bringing the real exchange rate back to its baseline level. Moreover, the lower inflation that prevails in the medium run pushes real interest rates upward and amplifies the fall in the output gap. Impulse responses to supply shocks illustrate the important role played by the supply block of this model at horizons that could matter for medium-run forecasts. For example, after a shock that gradually raises labor efficiency by 1% (by 0.95% after four years), real output increases by 0.6 percentage points over a 4-year horizon.

In a last section, we analyze monetary policy transmission in France through three policy experiments. In our first policy experiment, we assess the impact of a standard monetary policy shock, under three different expectations setups – agents may have model-consistent expectations on just financial variables (hybrid case), on both financial and non-financial variables (full MCE case) or neither, which corresponds to the VAR case. In all cases, FR-BDF shows a stronger response than our former model and this can be related to the widespread influence of the short term rate through expectations. There are some differences in outcomes that depend on how expectations are modeled. When non-financial variables are modeled in a forward-looking manner (full MCE case), this creates a dampening effect in comparison to the case where they would be backward looking (hybrid case). We can see this by the fact that the GDP response in the full MCE case is smaller than in the Hybrid case, with peak effects on GDP respectively around 0.1% and 0.3%. If we only model financial variables in a forward-looking way (hybrid case), we get on the contrary an amplification effect in comparison with the VAR-based case: peak effects on GDP are around 0.3% for the hybrid case and around 0.2% for the VAR-based case.

Second, with the model-consistent expectations version of the model, we can run alternative simulations in order to assess the macroeconomic impact in France of future policy shocks that would be announced and fully credible, such as forward guidance policies. With this version, we examine the impact in France of an announced cut in short-term interest rate for different numbers of quarters. The short run impact of these policies on long-term interest rates and exchange rates increases with the announced length of the shock. These stronger responses of financial variables lead to stronger responses of GDP and inflation. Stronger responses do not mean an explosive amplification: it appears that FR-BDF does not suffer from the forward guidance puzzle, i.e. the macroeconomic impact of forward guidance does not increase exponentially with the length of the shock, because of the way in which the future is discounted in the household consumption behavior. The peak effect on GDP increases almost linearly with the duration of the shock: after a shock announced to last 8 quarters, the peak effect on GDP is amplified by around 70% compared to the one obtained after a 1-quarter shock, which is slightly more than the double of the amplification obtained after a 4-quarter shock (around 30%).[4]

The third experiment we consider deals with how the asset purchase programmes (APP) conducted by the Eurosystem have affected the French economy. These programmes provide monetary accommodation by transferring liquidity directly to market participants in exchange for various assets. The empirical literature has identified effects on the term premium, but also on the exchange rate. We use these estimates to study the response of the French economy. We construct a counterfactual outcome for the French economy by removing the effects of APP on the term premium and exchange rates using estimates of shocks to these quantities stemming from APP. The simulations are conducted under VAR-based and model-consistent expectations, assuming that the French economy is otherwise disconnected from the rest of the euro area. Our main results show that APP had notable real and nominal effects on the French economy. Overall it would seem that APP is a shock that boosts exports and investment through a depreciation of the exchange rate and a fall in long interest rates. On the nominal side, it first results in imported inflation and then in domestic inflation due to an increase in factor costs. The main difference in outcomes due to the way in which expectations are modeled is in the response of inflation, which reacts much more strongly under model-consistent expectations.

The rest of the note is structured as follows. The next section provides a bird's-eye view of the overall features of FR-BDF. Section 3 discusses both expectations formation and the PAC framework. A detailed description of the different blocks of the model is presented in section 4. Section 5 is devoted to the model's main properties, while section 6 focuses on the transmission of standard and nonstandard monetary policy shocks. Section 7 concludes.

[4] In recent applications of DSGE models to the analysis of the consequences of forward guidance, a method of mitigating the puzzle consists of applying discounted Euler equations. See for example Nakata et al. (2019) and some examples that we provide in section 6.3.

2 Bird's-eye view of the model

This section gives an overview of the main blocks of FR-BDF. We describe the model as it would be applied for most types of policy analysis and unconditional simulations. For forecasting (e.g. Eurosystem macroeconomic projections) various model components would be exogenized to comply with the assumptions of those exercises (see last subsection).

2.1 Role of expectations in the presence of polynomial adjustment costs

For non-financial behavioral equations, the key role played by expectations is a consequence of applying the PAC framework, where, starting from an initial condition that differs from their long-term targets or after a shock that affects these long-term targets, agents adjust their decision variables toward the expected path of these long-term targets.

The agents' long-term targets – conceptually independent of the rest of the model – are determined by economic theory in a flexible economy, which is close to a neoclassical economy augmented with monopolistic competition. For example, firms' employment target, constructed from the production function and the associated first order condition, depends on aggregate demand and real labor costs. In some cases, some targets can directly depend on expected variables: for example, the consumption target depends on permanent income, defined as the expected average of future income flows.[5]

In the short run, economic agents seek to set the trajectory of a variable that they care about (e.g. employment) so as to minimize deviations from the long-term target (as described above) under PAC. Rearranging the first order condition of this problem leads to what are called the short run equations of FR-BDF, which describe the dynamics of the model's main variables given the dynamics of the target. A crucial feature of these short run equations is the presence of expectations regarding the target – variables that describe agents' beliefs regarding the future state of the economy. These expectations arise out of the adjustment costs – given their expected future state of the economy, agents know that their choices in the future are quite possibly different from today's choices; these constraints imply that it is prudent to adjust today's choices somewhat towards their future choices. For example, for a recessionary shock that would be expected to be temporary, the expected employment target would not fall as much and firms would cut fewer jobs (labor hoarding) than they would for shocks expected to be persistent.

In the financial block, adjustment costs play no role but expectations still appear through no-arbitrage conditions, namely the term structure of interest rates and the uncovered interest rate parity condition used for modeling exchange rates.

Figure 2.1.1 presents a simplified diagram of our model where variables directly affected by expectations appear in red, demonstrating how expectations play a widespread role in the whole model, i.e. in most of the non-financial and financial blocks.

[5]One exception is the investment target which does not necessarily set capital services equal to their flexible level in the short run (see subsection 4.3).

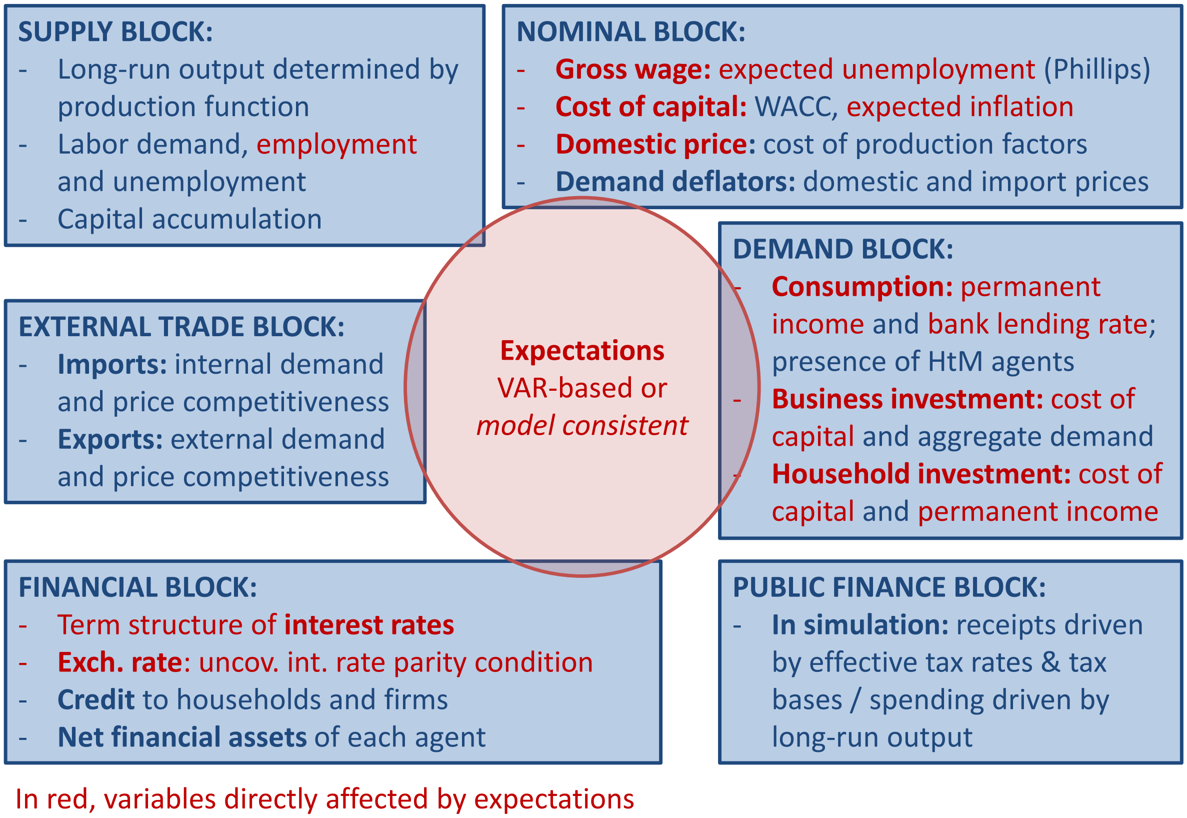


Figure 2.1.1: Simplified scheme of FR-BDF showing the widespread role of expectations

2.2 Expectations formation

In FR-BDF there are three types of expectations formation: (i) backward-looking expectations based on a satellite model called "E-SAT" (also referred to as VAR-based expectations), (ii) model-consistent expectations (MCE), (iii) a combination of these two (hybrid expectations). In the first case, agents base their expectations on a smaller and simpler model which is supposed to capture the main features of the economy (represented by the FR-BDF model). In the MCE case, agents are forward-looking and their predictions coincide with the FR-BDF forecast: they behave in a forward-looking manner with perfect foresight, i.e. they have a perfect knowledge of the future dynamics of the models' exogenous variables. Moreover, we are able to consider hybrid expectations. For example, Bernanke et al. (2019) performed simulations of FRB/US with MC expectations for financial agents, assumed to be well-informed, and VAR-based expectations for non-financial agents, assumed to have more limited information, in order to study the consequences of several monetary strategies.

E-SAT is a structural VAR model estimated with Bayesian methods. Its core is formed by IS and Phillips curves for the euro area and France and a Taylor rule for the euro area. To ensure consistency between E-SAT and the full model, we also make a strong simplification that France does not influence the euro area. The French IS curve relates the French output gap to the interest rate set by the governing council of the ECB. In order to take into account the sensitivity of economic developments in France to those in the euro area, this IS curve also relates the French output gap to the euro area output gap in a reduced form.

The model is completed by equations describing the behavior of 3 anchor variables. In the long run, inflation (both in France and in the euro area) and the interest rate are assumed to converge toward these long-term anchors. These long run anchors are measured either using survey-based or market-based expectations.

E-SAT can also be applied to compute expectations for variables in FR-BDF that are not within the previously described core. In this case, E-SAT has to be augmented with auxiliary equations that describe how the variable(s) of interest are related to the core variables.[6] As an example, business investment is dependent on an expectation concerning the deviation of market value added from its trend. The auxiliary equation relates the deviation of current value added to its lag and the output gap, which is in the E-SAT core.

In practical terms, contrary to what is usually done in the DSGE literature, equations are solved forward and agents build their expectations about infinite sums of variables of interest from the next period to the long run. In most cases, these sums are discounted. In each short run equation, the discounted sum of expected changes of the long-term target is computed using the PAC framework. In such cases, subjective discount factors of agents are adjusted to take into account the presence of adjustment costs. For example, when adjusting employment is very costly, firms will be more sensitive to future economic outlook relative to the current one for deciding the current level of their employment.

Finally, in the VAR-based case expectations appear as policy functions obtained by inverting the VAR model. In the MCE case expectations enter the model as they are defined using a forward-looking recursive form, i.e. with a finite number of leads of the variable itself.

2.3 Supply, value added price and labor market

We define long run output as the level of production that could be reached with full-utilization of the capital stock and the exogenous long run level of the unemployment rate. It is determined with a Constant Elasticity of Substitution (CES) production function of firms, which we proxy with the value added of market branches. This production function uses capital and labor as inputs with exogenously growing labor-augmenting technical progress.

In the short run, output is determined by demand components, through the clearing condition of the market of domestic goods and services. Capital services are determined by a standard accumulation equation that depends on investment which is determined in the long run by firms' first order condition with respect to capital (described in Section 4.3).

The value added price set by firms is modeled with a PAC equation. The long run price target is determined by a price frontier consistent with the CES production function, labor-demand condition and a markup stemming from monopolistic competition.

Labor demand is determined in the long run by firms' first order conditions with respect to labor, while polynomial adjustment costs smooth short run dynamics consistently with

[6] Note that the auxiliary equations are estimated separately and the core E-SAT coefficients stay unchanged.

labor hoarding observed on French data. Labor supply is determined by a wage Phillips curve *à la* Gali (Gali et al., 2011) that mainly relates wage growth to the expected unemployment gap. In the long run, this curve is vertical and the unemployment rate is set equal to its exogenous long run level.

2.4 Demand components and their deflators

The target of the main demand component, household consumption, is determined by permanent income and by an interest rate gap. Permanent income is constructed using a high discount rate that arises from a combination of household risk aversion and income uncertainty. Short run dynamics are determined via an estimated PAC equation that is extended with a term representing the current output gap, in order to take into account the sensitivity of hand-to-mouth consumers to current income.

Households formulate their investment target based on the same permanent income term as for consumption. Their decision also depends on the user cost of household capital and on relative prices of new and existing housing. This price sensitivity reflects their higher incentive to invest when the price of old housing is at a high level relative to the price of new housing.

The target for business investment is obtained through a steady-state investment-output ratio derived itself from first order conditions of the firms' optimization problem. It relates desired investment to output and to the user cost of capital. In the short run, following the standard Tobin's Q theory of investment, the presence of adjustment costs, polynomial in our framework, creates some stickiness of investment with respect to its target dynamics. Moreover, this PAC equation is extended with an *ad hoc* term – current value added growth – which improves the fit of the equation, one possible reason for this being the role of liquidity-constrained firms in investment dynamics.

As regards external trade, the target of real imports (both energy and non-energy) is mainly driven by the real exchange rate and an import intensity-adjusted measure of aggregate demand for imports, which in turn is determined as a weighted sum of the other demand components, including exports, with weights equal to their import content shares. For real exports, the target is mainly driven by world demand and the real exchange rate. The short run dynamics of real imports and exports are governed by ECM equations instead of following the PAC framework. These equations relate the growth rates imports and exports to the long run real growth rate of the economy and to either internal or world demand, along with an error correction component, i.e. the difference between the lagged level of the appropriate volume and its target.

Demand deflators are modeled with simple error-correction equations, with targets defined as weighted averages of domestic and import prices, the domestic price being the value added (VA) price of market branches and the import price being mainly determined by the price of competitors and the oil price.

2.5 Financial block

The short rate (3 month Euribor) is the main interest rate of the model – variation in this rate causes variation in the long-term government bond rate, which causes variation in the private interest rates. In simulation, it is based on either a simple AR(1) – with a long run anchor given by the historical mean – in order to abstract from issues related to the euro area, or on a Taylor rule that reacts to euro area inflation and output gap.

The 10-year government bond rate has an important role as the rate on which other long-term rates are based (long-term bank lending rates, as well as the corporate bond rate and cost of equity). In simulation, it is determined with a term structure equation – the long rate is simply the sum of an expectation component and a time-varying term premium.

Exchange rates are determined via uncovered interest rate parity (UIP). Two exchange rates are computed: the rate between the euro and the US dollar and the rate between the euro and a basket of currencies used by France's other trading partners. These rates are then applied to determine the price of oil in dollars and the price levels of the France's competitors.

Finally, Figure 2.5.1 provides a simplified diagram of the FR-BDF financial block. First, the short-term rate influences all expectations, even in the VAR-based case. Second, arrows show the transmission channels, in particular the direct effect of the short-term rate on the long-term rate through the term structure and on exchange rates through the UIP condition. Third, the effect of interest rates on business investment transmits through the three components of the weighted average cost of capital (WACC), i.e. the bank lending rate, the bond rate and the cost of equity. Interest rates also influence households' consumption and investment choices through their bank lending rate.

Variables	Effect in FR-BDF
Short run interest rate	All expectations
Long run interest rate	All private interest rates
Bank lending rate households	Demand of households
Return rate of financial assets	Net financial income
Bank lending rate firms	Business investment
Bond rate of firms	Business investment
Cost of Equity	Business investment
Exchange rate	External trade

Figure 2.5.1: Simplified diagram of the financial block of FR-BDF

2.6 Accounting framework and public finances

The accounting framework of FR-BDF mimics the quarterly national accounts with great detail and is structured around two decompositions:

- A branch decomposition (market/non-market) of the value added account and of labor market variables and a product decomposition (energy/non-energy) in the specific case of imports;

- *Supply and use* accounts in value and volume, as well as sector accounts in value for five institutional sectors (firms, households, government, non-profit organizations and the rest of the world).

Consistency between these axes is ensured through bridge equations, i.e. production and labor market variables of sector accounts (value added and wages) are extrapolated with simple equations which relate them to corresponding variables from the branch decomposition. For example, the nominal value added of firms is extrapolated with a proportional factor with respect to the one of market branches.

The government block is particularly detailed and designed in such a way as to allow interactions with the public finance model of the Banque de France. In particular, for many variables of this block two modes were created: a forecasting mode, where many nominal variables are exogenized and come directly from the public finance model; a simulation mode, where these variables are endogenous and depend on exogenous effective tax rates or exogenous ratios. More precisely, in the simulation mode, we adopt the following common principles on the receipt and spending sides:

- On the receipt side, each receipt is determined by an exogenous effective tax rate and an endogenous tax basis;

- On the spending side, some spending variables are directly related to macroeconomic aggregates with effective rates. For example, unemployment benefits are directly related to unemployment and wage per capita. For other spending variables (excluding social transfers as explained below), e.g. intermediate consumption or government investment, the ratio of their volume relative to long run output is assumed to be exogenous.

In simulation, we endogenize a fiscal rule, which assumes that an instrument – social transfers in our baseline case – will be used by the government to ensure the convergence of its net asset ratio toward its long run target.

2.7 Long run of the model

In the long run, the variables of the model follow a balanced growth path, where, on the one hand, output growth is determined by labor efficiency and demography, and, on the other hand, inflation is determined by the ECB inflation target. When the price frontier and first order conditions with respect to labor and capital are met, the level of output endogenously converges toward the long run output (defined in the subsection describing the supply block). Contrary to closed-economy models, the real interest rate here is exogenous in the long run, determined by the exogenous nominal interest rate and by the ECB inflation target. Setting the growth rate of world demand and the inflation rates of competing countries equal to rates at the domestic level for ensuring a balanced growth path, the convergence of demand toward the long run output determines, in our small open economy framework, the long run equilibrium of the real exchange rate, a key driver of external trade (as shown in section 4.6). As competitors' prices are exogenous, as well as the nominal exchange rate in the long run, the level of the real exchange rate determines the level of the domestic value added price in the long run.

2.8 Estimation

The estimation of equations requires specifying the way in which agents form their expectations. As in FRB/US, we choose here to estimate our model under VAR-based expectations, in order to be able to estimate the model block by block. Although this might create simultaneity biases, this approach seems more suited to such a large-scale model for two reasons. First, this approach has the advantage that potential misspecifications of some blocks will not spoil the whole estimation of the model, as would be the case with joint estimation. Second, this makes the estimation more flexible: a joint estimation would be very difficult and the cost of conducting a fully fledged joint re-estimation each time a single block is revised would be too large. Our backward-looking expectations stem from our structural Bayesian VAR. This VAR is used to generate the various expectation terms of the model. In particular, we estimate short run PAC equations with an iterative OLS procedure *à la* FRB/US which iterates two steps: the estimation of short run coefficients conditional on expectation terms; the computation of expectation terms with the VAR and conditional on these coefficients. We provide more details about estimation in Section 4.1.

3 Expectation formation and the polynomial adjustment costs framework

In this section, we explain the way in which agents form their expectations and the reasons why these expectations play a role in the model through the present value of future changes in targets in most short run equations, due to the presence of polynomial adjustment costs. This second topic is split into two subsections devoted to the micro-foundations of the polynomial adjustment costs framework, on the one hand, and the computation of present values, on the other.[7]

3.1 Expectations formation

Expectations are at the core of modern macroeconomic models as economic decisions (consumption, investment, etc) depend on future conditions. The better expectations are incorporated the more reliable is the model. Expectations are supposed to dampen or amplify the impact of certain shocks: e.g. to dampen the employment response in the case of a temporary negative demand shock or to amplify the response of financial variables when a monetary policy shock is announced to last longer than expected. The presence of expectations is one of the main advantages of FR-BDF with respect to the former forecasting model of the Banque de France (Mascotte). This feature of FR-BDF enables the study of questions connected to monetary policy or macro-financial issues.

There are two types of expectations formation in FR-BDF. The first type, our baseline case described in this section, assumes that agents have limited knowledge about the model. They form their expectations based on a simpler model containing much fewer variables, which we refer to as the Expectation SATellite model (E-SAT). These expectations are backward-looking and referred to as VAR-based expectations because in this case expectations are identical to the forecast of a constrained VAR model. The second type of expectations formation, described in section 6.1, is forward-looking: agents have full knowledge about the model and they can adjust their decisions to information about the future the moment they receive it. These expectations are referred to as "model-consistent".

In some practical experiments (see section 6) we use a hybrid framework in which financial agents form their expectations using the second method while others have limited information to take their decisions and base their expectations on a simple VAR model. We believe that this is an interesting exercise as there is still no consensus about how agents form their expectations. For example, one may argue that agents making financial decisions would prefer to take into account any available information about the economy even if it is costly, unlike a decision on short run consumption.

FR-BDF contains 11 variables for which agents form expectations. Some of them appear in the short run behavioral equations of non-financial agents, due to the presence of adjustment costs. This concerns the short run equations of household consumption and investment, corporate consumption, employment, wage and value added price. Other expectation variables (permanent income and inflation) appear directly in the long run equations of the

[7] Please note that this section treats only backward-looking expectations, i.e. based on a small VAR. Model-consistent expectations are discussed in 6.1.

desired targets (household consumption and corporate investment respectively). Finally, expectation terms are found in asset pricing equations which are derived from no-arbitrage conditions as well as in simple discounting schemes: domestic and foreign short run interest rates.[8]

In what follows we describe the FR-BDF VAR model that agents use to form their expectations when they do not have model-consistent expectations. For the details about how we compute the present values of expected variables, we invite the reader to consult section 3.3.

3.1.1 Specification of the expectation satellite model

The expectation satellite model E-SAT consists of a core, with eight equations and an augmented part which varies depending on blocks to be estimated.

Semi-structural equations and VAR representation The core of E-SAT is based on a small model with two blocks: France and the euro area.[9] The model consists of the IS and Phillips curves for each block and is completed with a Taylor rule which sets the interest rate as a function of EA inflation and its output gap. This model is a structural VAR model in the spirit of Rudebusch & Svensson (1999), where we add shifting endpoints of inflation and the interest rate. As shown in Kozicki & Tinsley (2001), taking into account shifts in endpoints supports a more substantial term structure role for short rate expectations. The core E-SAT model has eight equations:

$$\begin{aligned}
(1-\lambda_q L)\hat{y}_t &= -\sigma_q(i_{t-1} - \pi_{Q,t-1} - \bar{\imath}_{t-1} + \bar{\pi}_{t-1}) + \delta_q \hat{y}_{ea,t} + \varepsilon_{q,t} \\
(1-\lambda_\pi L)(\pi_{Q,t} - \bar{\pi}_t) &= \kappa_\pi \hat{y}_{t-1} + \varepsilon_{\pi,t} \\
(1-\lambda_i L)\,(i_t - \bar{\imath}_t) &= (1-\lambda_i)(\alpha_i(\pi_{ea,t-1} - \bar{\pi}_{t-1}) + \beta_i \hat{y}_{ea,t-1}) + \varepsilon_{i,t} \\
(1-\lambda_{q,ea} L)\hat{y}_{ea,t} &= -\sigma_{q,ea}(i_{t-1} - \pi_{ea,t-1} - \bar{\imath}_{t-1} + \bar{\pi}_{ea,t-1}) + \varepsilon_t^{q,ea} \\
(1-\lambda_{\pi,ea} L)(\pi_{ea,t} - \bar{\pi}_{ea,t}) &= \kappa_{\pi,ea} \hat{y}_{ea,t-1} + \varepsilon_t^{\pi,ea} \\
(1-\lambda_{\bar{\imath}} L)(\bar{\imath}_t - \bar{\imath}) &= \varepsilon_{\bar{\imath},t} \\
(1-\lambda_{\bar{\pi}} L)(\bar{\pi}_t - \bar{\pi}) &= \varepsilon_{\bar{\pi},t} \\
(1-\lambda_{\pi,\bar{e}a} L)(\bar{\pi}_{ea,t} - \bar{\pi}_{ea}) &= \varepsilon_{\bar{\pi},ea,t}
\end{aligned}$$

where L stands for the lag operator. The IS curve of each sector relates the output gap to the real interest rate gap (with respect to the shifting endpoint $\bar{\imath}_t$ of the interest rate). We also take into account a co-movement between the French output gap $\hat{y}_t$ and that of the euro area $\hat{y}_{ea,t}$ through a global demand (trade) channel and not only through the ECB's monetary policy. The Phillips curve relates domestic inflation $\pi_{Q,t}$ to the inflation shifting endpoint $\bar{\pi}_Q$ and the output gap. The inertial Taylor rule relates the interest rate i_t to its lag, euro area inflation $\pi_{ea,t}$ and the euro area output gap $\hat{y}_{ea,t}$.

[8]Here we have listed 11 expectation variables. In order to obtain 11, we take into account that we have two types of present values of domestic short run interest rate: a discounted present value and a non-discounted one.

[9]We do not consider any other foreign variables since: (i) we are limited in the number of variables within ESAT, and (ii) EA variables may be viewed as an approximation for the rest of the world.

We have also considered an alternative specification of the ESAT model, where the main stabilization mechanism is ensured through a real exchange rate adjustment: a Taylor rule is replaced with an AR(1) process of the real interest rate, and an IS curve is augmented by an additional term $log(p_t) - log(p_{ea,t}) + log(e_t)$, where e_t is an exchange rate for euro area countries and is supposed to vary around 1. This model is perfectly consistent with the assumptions of FR-BDF, but the convergence of that specification is much longer than in the baseline.[10]

Forming the A and B matrices for applications We can represent the E-SAT model as a structural VAR of a vector Z_t of eight variables and an intercept: $AZ_t = BZ_{t-1} + \varepsilon_t$. The A and B matrices have dimension 9×9. We can deduce from this structural form the following reduced form:

$$Z_t = HZ_{t-1} + \eta_t \tag{1}$$

with $H = A^{-1}B, \eta_t = A^{-1}\varepsilon_t$. Finally, the reduced form directly delivers the following i-step ahead forecast: $Z^e_{t+i} = H^i Z_t$.

Given the following order of variables in vector $Z = [1, \hat{y}, i, \pi_Q, \hat{y}_{ea}, \pi_{ea}, \bar{\imath}, \bar{\pi}, \bar{\pi}_{ea}]$, the expressions of A and B are easily deduced from the equations of the model:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\delta_q & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_q & -\sigma_q & \sigma_q & 0 & 0 & \sigma_q & -\sigma_q & 0 \\ 0 & 0 & \lambda_i & 0 & \beta_i(1-\lambda_i) & \alpha_i(1-\lambda_i) & -\lambda_i & 0 & -\alpha_i(1-\lambda_i) \\ 0 & \kappa_\pi & 0 & \lambda_\pi & 0 & 0 & 0 & -\lambda_\pi & 0 \\ 0 & 0 & -\sigma_{q,ea} & 0 & \lambda_{q,ea} & \sigma_{q,ea} & \sigma_{q,ea} & 0 & -\sigma_{q,ea} \\ 0 & 0 & 0 & 0 & \kappa_{\pi,ea} & \lambda_{\pi,ea} & 0 & 0 & -\lambda_{\pi,ea} \\ (1-\lambda_{\bar{\imath}})\bar{\imath} & 0 & 0 & 0 & 0 & 0 & \lambda_{\bar{\imath}} & 0 & 0 \\ (1-\lambda_{\bar{\pi}})\bar{\pi} & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{\bar{\pi}} & 0 \\ (1-\lambda_{\pi,\bar{e}a})\bar{\pi}_{ea} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_{\pi_{\bar{e}a}} \end{bmatrix}$$

These eight equations which form the core of the ESAT model are not always sufficient to describe the formation of agents' expectations, which may bear on other variables, absent from E-SAT. This will require adding auxiliary equations into the system. Assume for instance that the model includes expectations on the future values of a target variable n^*_t. In this case agents have to foresee the changes in the trend of n^*_t, denoted $\bar{n}^*_t$, and the changes in its cyclical component $\hat{n}^*_t$. We then add to E-SAT two auxiliary equations, which may be,

[10]Details are available upon request.

for instance

$$\left(1-\lambda_{\hat{n}*}L\right)\hat{n}_t^* = a_{\hat{n}*}\hat{y}_{t-1} + b_{\hat{n}*}\left(i_{t-1}-\bar{\imath}_{t-1}\right) + c_{\hat{n}*}\left(\pi_{Q,t-1}-\bar{\pi}_{Q,t-1}\right) + \varepsilon_{\hat{n}*,t}$$
$$\Delta\bar{n}_t^* = \Delta\bar{n}_{t-1}^* + \varepsilon_{\bar{n}*,t}$$

With these two added variables vector Z_t becomes

$$Z_t = (1, \hat{y}_t, i_t, \pi_{Q,t}, Q_{ea,t}, \pi_{ea,t}, \bar{\imath}_t, \bar{\pi}_{Q,t}, \bar{\pi}_{ea}, \hat{n}_t^*, \Delta\bar{n}_t^*)'$$

Matrices A and B are now 11x11. The added rows of A are:

$$A_{10} = \left[\begin{array}{ccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$
$$A_{11} = \left[\begin{array}{ccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Those of B are

$$B_{10} = \left[\begin{array}{ccccccccccc} 0 & a_{\hat{n}*} & b_{\hat{n}*} & c_{\hat{n}*} & 0 & 0 & -b_{\hat{n}*} & -c_{\hat{n}*} & 0 & \lambda_{\hat{n}*} & 0 \end{array}\right]$$
$$B_{11} = \left[\begin{array}{ccccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Finally, the additional columns - above rows 10 and 11 - of A and B are simply filled with zeros, as the core variables are assumed to be unaffected by the auxiliary variables.

3.1.2 Estimation of the core of the expectation satellite model

Methodology We estimate the core system (the first five equations of the system written in deviation from long run anchors, in subsection 3.1.1) using Bayesian techniques and detrended data. The choice of applying Bayesian estimation instead of maximum likelihood is driven by two factors: a relatively small sample and potential misspecification of the model.

The sample that we use to estimate E-SAT is not very large and the data might therefore not contain sufficient information to identify (with precision) the parameters. When there is a lack of identification, the likelihood function is flat in certain directions. In this case the Bayesian approach is more efficient and less sensitive to sample size, since priors can be used to introduce "curvature" into the objective function. From a numerical perspective maximizing the posterior is "easier" than maximizing the likelihood function. The prior information may also be helpful to discriminate between hypotheses.

Sample size is not the only problem that we have to deal with. It is difficult to match euro area inflation with a simple Phillips curve.[11] This is due to the fact that all E-SAT variables enter the policy functions of expected terms, and more complicated policy functions are harder to interpret during analysis. When we estimate E-SAT with maximum likelihood (ML), depending on the initial value of the estimation, the α_i weight of inflation in the Taylor rule is either negative, or below one and not significant.[12] Due to the stylized and often misspecified nature of DSGE (VAR) models, the likelihood is known to often peak in regions of the parameter space that are contradictory with common observations, leading to the "dilemma of absurd parameter estimates" (Adjemian & Pelgrin (2008), Schorfheide

[11] For example in the last versions of FRB/US, in order to recover parameters of the inflation equation, wage and price Phillips curves are estimated jointly.

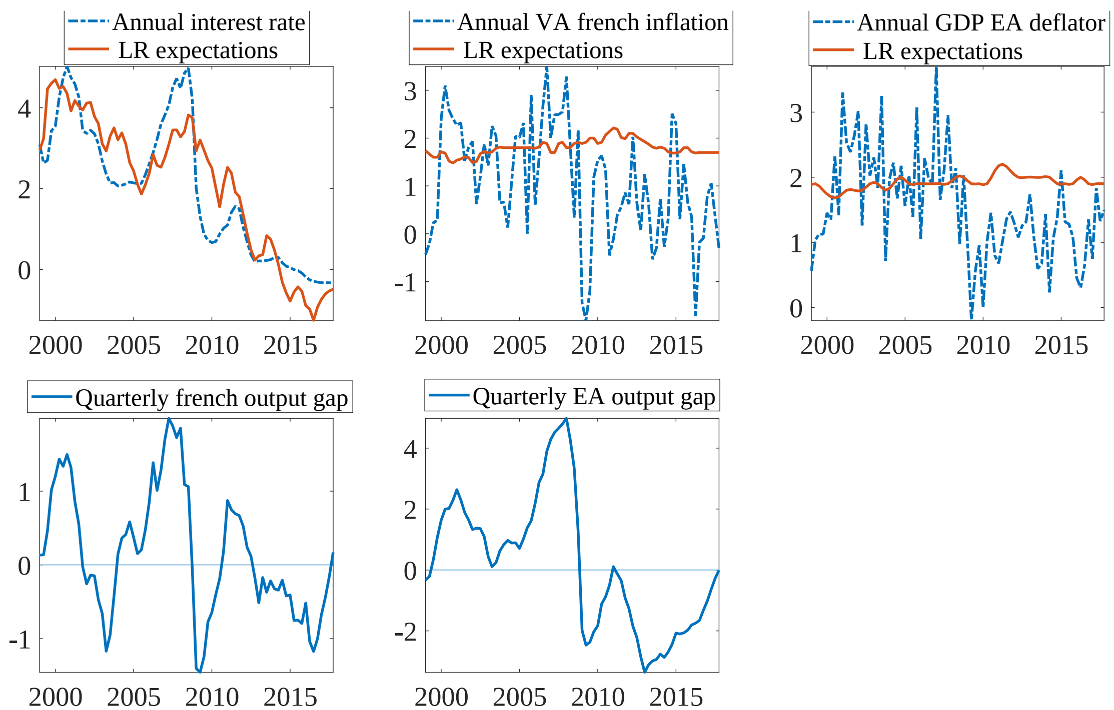
[12] We also tried to reduce the E-SAT to a model of the euro area economy of three equations and to exclude the zero lower bound region, but the results remained unchanged.

(2011)). α_i was not the only parameter that we had difficulties estimating with the ML method. The parameters σ_q, $\sigma_{q,ea}$ and β_i also suffered from the misspecification.

The priors are specified in Table 3.1.1. We are aware of the criticism levelled at the Bayesian approach saying that by "playing" with priors we may impose a desirable outcome. In order to address this concern, we ensured that our results were robust with a different specification for our priors, which we always chose so as to stay within the boundaries of conventional wisdom.

Some of the parameters of the equations of the long run anchors are calibrated and others are estimated, see table 3.1.3. The steady state of annualized inflation in France and the euro area are set to the ECB target: 1.9%. The persistence $\lambda_{\bar{i}}$ of the long run anchor of the interest rate has strong implications for the term premium. For the reasons explained in section A.2 we calibrate this parameter. Other parameters are estimated using OLS equation by equation on a pre-crisis sample: 1999Q1 - 2008Q4.

Figure 3.1.1: Time series used for the estimation of E-SAT



Data The estimation period is 1999Q1-2017Q4 and time series are shown in Figure 3.1.1. Below we provide the description of data construction and its sources.

- The French output gap is calculated as a difference between actual and long run value added.[13]

[13]To estimate E-SAT, we use a vintage of the output gap estimate which is slightly different from the

Table 3.1.1: Priors and estimation results

	Prior distribution	Mode	Posterior distribution St Dev	Mean	10%	90%
$\boldsymbol{\sigma_q^\varepsilon}$	Inv. Gamma (0.1,2)	0.33	0.03	**0.34**	0.29	0.39
$\boldsymbol{\sigma_{pi}^\varepsilon}$	Inv. Gamma (0.1,2)	0.26	0.02	**0.26**	0.23	0.30
$\boldsymbol{\sigma_{\pi,ea}^\varepsilon}$	Inv. Gamma (0.1,2)	0.18	0.01	**0.19**	0.16	0.21
$\boldsymbol{\sigma_i^\varepsilon}$	Inv. Gamma (0.1,2)	0.09	0.01	**0.10**	0.08	0.11
$\boldsymbol{\sigma_{q,ea}^\varepsilon}$	Inv. Gamma (0.1,2)	0.56	0.05	**0.58**	0.50	0.66
$\boldsymbol{\delta}$	Normal(0.2,0.2)	0.08	0.03	**0.08**	0.03	0.12
$\boldsymbol{\lambda_q}$	Beta(0.5,0.2)	0.73	0.06	**0.73**	0.62	0.84
$\boldsymbol{\lambda_{q,ea}}$	Beta(0.5,0.2)	0.94	0.03	**0.93**	0.89	0.97
$\boldsymbol{\sigma_q}$	Normal(0.6,0.15)	0.27	0.10	**0.28**	0.11	0.45
$\boldsymbol{\sigma_{q,ea}}$	Normal(0.6,0.15)	0.54	0.13	**0.54**	0.33	0.76
$\boldsymbol{\lambda_\pi}$	Beta(0.5,0.2)	0.58	0.09	**0.58**	0.43	0.72
$\boldsymbol{\lambda_{\pi,ea}}$	Beta(0.5,0.2)	0.34	0.10	**0.35**	0.19	0.51
$\boldsymbol{\kappa_\pi}$	Gamma(0.5,0.2)	0.07	0.03	**0.08**	0.02	0.13
$\boldsymbol{\kappa_{\pi,ea}}$	Gamma(0.5,0.2)	0.04	0.01	**0.04**	0,.2	0.05
$\boldsymbol{\lambda_i}$	Beta(0.3,0.15)	0.93	0.03	**0.92**	0.88	0.97
$\boldsymbol{\alpha}$	Normal(1.5,0.25)	1.22	0.28	**1.19**	0.71	1.66
$\boldsymbol{\beta}$	Gamma(0.2,0.1)	0.07	0.04	**0.09**	0.02	0.16

- The euro area output gap and euroa area inflation are taken from the Eurosystem's forecasting exercise from March 2018. The euro area inflation is computed with the GDP deflator.

- The French inflation rate is measured as the quarterly growth rate of the value added deflator.

- The interest rate is the 3-month Euribor, interpolated with Eonia between 1995Q2 and 1999Q4.

- Long run expectations of both inflation rates come from the long run professional consensus forecast surveyed by the private company "Consensus Economics". The long run horizon is here an average of horizons going from 5 to 10 years. The long run expectation of euro area inflation is retropolated before 2002 with a weighted average of five countries: France, Italy, Germany, the Netherlands and Spain.

one used in the rest of the model. As detailed in section 4.3, long run value added of market branches is calculated using a measure of capital services for which we need wealth accounts. Because wealth accounts in base 2014 were not available at the time of the estimation of E-SAT, we used wealth accounts in base 2010 after having re-based the deflators of capital and investment sub-components in 2014. Still, we checked that the last version of the output gap would not have substantially changed these results if we had used the wealth accounts in base 2014.

Table 3.1.2: Posterior mean variance decomposition (in %)

	$\boldsymbol{\sigma_i^\varepsilon}$	$\boldsymbol{\sigma_{pi}^\varepsilon}$	$\boldsymbol{\sigma_q^\varepsilon}$	$\boldsymbol{\sigma_{\pi,ea}^\varepsilon}$	$\boldsymbol{\sigma_{q,ea}^\varepsilon}$
$\hat{y}_t$	39.7	6.26	31.92	1.72	20.4
$(\pi_{Q,t} - \bar{\pi}_t)$	11.25	77.76	5.48	0.5	5.02
$(i_t - \bar{\imath}_t)$	67.35	0	0	7.32	25.33
$\hat{y}_{ea,t}$	38.52	0	0	2.84	58.64
$(\pi_{ea,t} - \bar{\pi}_{ea,t})$	11.83	0	0	73.48	14.69

- The long run expectation of the interest rate comes from 5-year futures of the 3-month Euribor.

The long run expectation of the interest rate is shifted by a constant to make it share the same average as the 3-month Euribor.[14]

Table 3.1.3: Parameters of the long run expectations

Estimated		Calibrated	
$\bar{\imath}$	3.68	$\lambda_{\bar{\imath}}$	0.985
$\lambda_{\pi_{ea}}$	0.93	$\bar{\pi}$	1.9
λ_π	0.93	$\bar{\pi}_{ea}$	1.9

$\bar{\imath}$, $\bar{\pi}$ and $\bar{\pi}_{ea}$ are annual rates in p.p.

3.1.3 Estimation results and impulse responses

We use the mean of the posterior distribution as the point estimate of parameters in FR-BDF. All the parameters (except $\sigma_{q,ea}$) seem to be well identified and the posterior distribution differs from our priors, see Figure A.1 in section A.3.[15] The values of parameters confirm conventional wisdom. They also meet the stability conditions of the core block of E-SAT described in appendix A.1. As regards the disturbances, as shown by the posterior mean variance decomposition (Table 3.1.2), the interest rate shock seems to play an important role in explaining the dynamics of the French and the euro area output gap.

In this subsection, we look at the response of the output gap and inflation to shocks to two variables: the interest rate and the euro area output gap. For computing impulse responses, we also use the mean of the posterior distribution as a central tendency.[16] We

[14]In the current version we decided not to shift the long run expectations of French and euro area inflation rates, i.e. to assume that the long run expectation for value added inflation would be the same as for HICP inflation.

[15]Identification of parameters is a vague notion. Not all of the parameters are well identified "globally", irrespective of the estimation approach. It seems that without priors, the posterior distribution is rather flat. When we used the ML method we encountered a problem estimating three parameters: α_i, δ_q and σ_q. Their values neither have a conventional signs nor are they significant.

[16]We also checked the impulse response functions (IRFs) computed using the mode of the posterior distribution and the results were unchanged.

find that in both cases the impulse responses have humped shapes, with expected signs and responses close to zero after around 20 years.

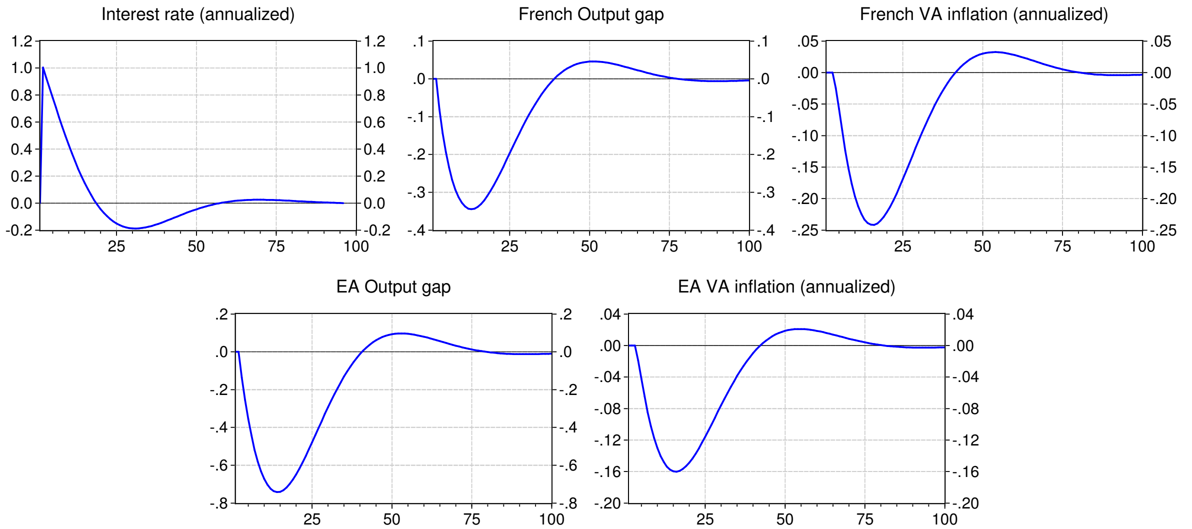


Figure 3.1.2: Impulse responses for E-SAT of French and euro area variables to an interest rate shock

Response to an interest rate shock First, we look at the properties of key impulse responses of the satellite model to a 0.25pp interest rate hike (i.e. 1pp for an annualized rate), with a historical persistence of 0.93 (Figure 3.1.2). In the short run, this hike has a direct negative impact on both output gaps and, hence, on inflation in the euro area as well as in France. The fall in inflation amplifies the rise in the real rate and, hence, the fall in demand due to the Taylor rule inertia. This generates a hump after around three years, with a trough in the output gap at -0.34pp and of value added inflation (annualized) at -0.24pp: obviously, the Phillips curve slope is rather steep: $\kappa_\pi/(1-\lambda_\pi)\cdot 4 = 0.076/(1-0.43)\cdot 4 = 0.71$ in annual terms. This estimate is higher than 0.5 reported in Chatelais et al. (2015). As the interest rate shock is temporary, the output gap and inflation return to their steady state after this hump.

The sensitivity of the euro area output gap with respect to the change in the real interest rate seems to be much stronger than in the case of France. This is consistent with the estimated $\sigma_{q,ea}$ which is almost twice as high as σ_q and also higher than the persistence of euro area output gap, see Table 3.1.1.

The reaction of the euro area inflation is much smaller than that in France. The implied Phillips curve slope is 0.22 in annual terms.

If we compare these results with those of the workhorse VAR study of Peersman & Smets (2001) based on euro area data, we find that our results are not far from theirs. If we rescale their shock to 100 bp on the annualized interest rate, the impact after 12 quarters is -0.5% for output and -0.3% for consumer prices. Regarding output, our response is slightly weaker. As for consumer prices, to facilitate the comparison, we look at the cumulative response of

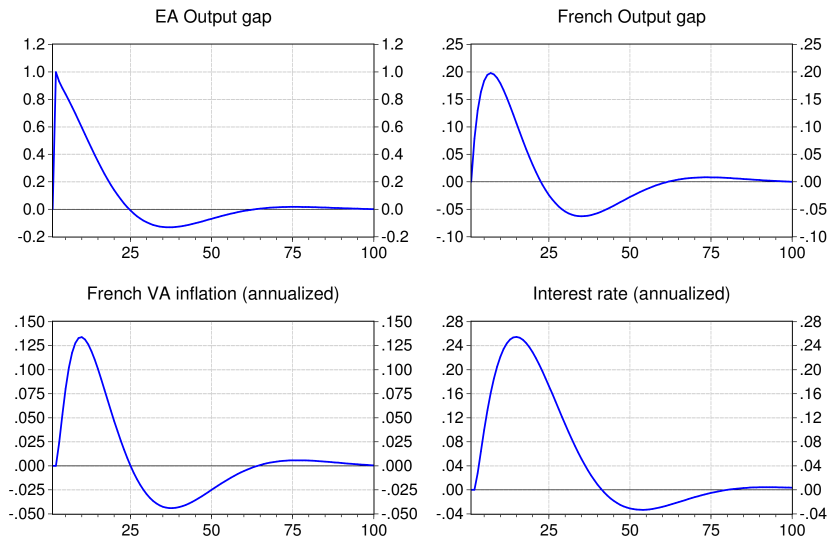


Figure 3.1.3: Impulse responses for E-SAT of French and euro area variables to a shock to euro area output gap

inflation and obtain an impact on the value added price level of -0.5pp after 12 quarters, which is stronger than theirs.

Response to a shock to the euro area output gap Here, the dynamics of the variables are triggered through two channels: the trade channel and the monetary policy channel, see Figure 3.1.3. Due to the trade channel included in the French IS curve ($\delta\hat{y}_{ea}$), we obtain a positive spillover effect of the euro area shocks to France.

3.2 Microfoundations for the Polynomial Adjustment Costs framework

The foundation of the Polynomial Adjustment Costs framework (PAC) – which we use to model most of our non-financial behavioral equations – is an extension of ideas familiar to many from the Dynamic Stochastic General Equilibrium (DSGE) literature. The framework can be characterized as a generalization of the modeling devices used in e.g. Rotemberg pricing (Rotemberg, 1982), where firms seek to minimize the deviation between the target price of the good they are selling and the actual price they set today under quadratic costs of adjustment. The PAC framework takes this idea further by making the cost function polynomial: not only is it quadratically costly to deviate from the target, but also the m latest differences are penalized in the PAC cost function. Furthermore, the PAC framework also implies an error correction equation – augmented with an expected present value of future changes in the target – that can be obtained by rearranging the first order condition associated with the cost function. The derivation of the error correction equations is based on the outline of the PAC framework presented in Tinsley (2002).

This section is divided into two subsections. The first presents an outline of the derivation discussed in the previous paragraph, starting from a generic m-th order cost polynomial. The second describes the constraint that we use for ensuring growth neutrality.

3.2.1 Cost functions and rational error-correction equations

The starting point for deriving the error correction equations is a cost function that agents seek to minimize by choosing a sequence of values for their choice variable y_t

$$C_t = \sum_{i=0}^{\infty} \beta^i \left[\left(y_{t+i} - y^*_{t+i}\right)^2 - \sum_{k=1}^{m} b_k \left((1-L)^k \, y_{t+i} \right)^2 \right] \tag{2}$$

The cost function penalizes deviations of the decision variable y_t from its equilibrium value y^*_t (also interchangeably referred to as desired or target value) and from m differences. The order of the lag/lead polynomial is determined by the order of adjustment costs that the agents are assumed to be subject to, i.e. by m. In addition, β is a discount factor and b_k are cost parameters (also referred to as adjustment coefficients). According to Brayton et al. (2000), differentiation with respect to y_t yields

$$2\left(y_t - y^*_t\right) + \sum_{i=0}^{\infty}\sum_{k=1}^{m} \beta^i b_k \frac{\partial \left((1-L)^k \, y_{t+i} \right)^2}{\partial y_t} = 0$$

in which

$$\sum_{i=0}^{\infty} \beta^i b_k \frac{\partial \left((1-L)^k \, y_{t+i} \right)^2}{\partial y_t} = 2 b_k \left[(1-L)^k \left(1 - \beta F\right)^k y_t \right]$$

so that

$$\left(y_t - y^*_t\right) + \sum_{k=1}^{m} b_k \left[(1-L)\left(1-\beta F\right) \right]^k y_t = 0$$

which can also be expressed after some algebra as

$$A(L)A(\beta F)y_t - A(1)A(\beta)y^*_t = 0 \tag{3}$$

where $A(L)$ is a polynomial in the lag operator L and $A(\beta F)$ is a polynomial in the lead operator F, both of degree m. With $m = 1$, only the change in y_t is subject to adjustment costs with $A(L) = 1 - \alpha_1 L$ and $A(\beta F) = 1 - \alpha_1 \beta F$. The associated *decision rule* that is obtained by multiplying (3) by $A(\beta F)^{-1}$ and significant rearranging is

$$\Delta y_t = A(1)\left(y^*_{t-1} - y_{t-1}\right) + A(1)\sum_{i=0}^{\infty} (\alpha_1 \beta)^i \Delta y^*_{t+i}$$

within which terms can be rearranged and relabeled to obtain an expression that uses a more compact notation:

$$\Delta y_t = a_0 \left(y^*_{t-1} - y_{t-1}\right) + \sum_{i=0}^{\infty} d_i \Delta y^*_{t+i} \tag{4}$$

where $a_0 = d_0 = A(1)$ and d_i for $i > 0$ are transformations of the lag/lead polynomial A and the discount factor β.

The most ubiquitous case when this framework is applied in the FR-BDF model is where $m > 1$, with $m = 2$ being particularly common. Then $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 ... - \alpha_m L^m$, $A(\beta F) = 1 - \alpha_1 \beta F - \alpha_2 (\beta F)^2 ... - \alpha_m (\beta F)^m$ complicating the steps needed to obtain an expression similar to (4).[17] It can then be shown that when $m > 1$, the analogue of (4) is

$$\Delta y_t = a_0 \left(y_{t-1}^* - y_{t-1} \right) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \sum_{i=0}^{\infty} d_i \Delta y_{t+i}^* \tag{5}$$

with $a_k = -\sum_{j=k+1}^{m} \alpha_j$, $k > 0$.

To a casual observer (5) might look like an error-correction equation with the additional term $\sum_{i=0}^{\infty} d_i \Delta y_{t+i}^*$ - the discounted sum of changes in the target – with an unclear role. One intuitive interpretation is to think of the term as a correction factor that tries to accommodate already today for the fact that the target itself is moving and will not be the same tomorrow. If the target was expected to remain constant at its last value, the agent would not care about adjustment costs in the future and the sum would be zero. In the standard case this will not be true as the target will move. Thus, the agent knows that further adjustments will be necessary in the future – even if they are exactly on target today – and that these changes will be costly. Hence, in order to minimize total costs some additional adjustments will be reasonable even today.

Note that

$$d_i = A(1) A(\beta) \iota_m' \left[1 - G \right]^{-1} G^i \iota_m \tag{6}$$

where ι_m is a $m \times 1$ vector with the mth element equal to one, zeroes elsewhere and G is the $m \times m$ matrix

$$G = \begin{bmatrix} 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 1 \\ -a_m \beta^m & -a_{m-1} \beta^{m-1} & \dots & -a_{m-j} \beta^{m-j} & -a_1 \beta \end{bmatrix} \tag{7}$$

3.2.2 Constraint used for ensuring growth neutrality

In order to obtain a balanced growth path in the long run, where actual variables would be equal to their targets ($y_t = y_t^*$), an important condition that needs to be met is that all equations are growth neutral. In practice this means that the sum of the a_k and d_i coefficients is restricted to equal unity when the model is estimated. To see why this is appropriate, assume that the system is in a balanced growth equilibrium, so that $\Delta y_t = \Delta y_t^* = g$. Substituting this into (5) yields

$$a_0 \left(y_{t-1}^* - y_{t-1} \right) = \left[1 - \sum_{k=1}^{m-1} a_k - \sum_{i=0}^{\infty} d_i \right] g \tag{8}$$

[17] See Brayton et al. (2000) and Tinsley (2002) for details.

from which it is clear that $y^*_{t-1} - y_{t-1} = 0$ if and only if $\left[1 - \sum_{k=1}^{m-1} a_k - \sum_{i=0}^{\infty} d_i\right] = 0$. Note that the term $\sum_{i=0}^{\infty} d_i = \omega$ is the share of the nonstationary component of the expected changes in the target.

In practice, when error correction equations of the form (5) are estimated, the constraint ensuring stationarity is imposed with the addition of a term resembling the right hand side of (8) into any such equation[18]

$$\Delta y_t = a_0 \left(y^*_{t-1} - y_{t-1}\right) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \sum_{i=0}^{\infty} d_i \Delta y^*_{t+i} + \left[1 - \sum_{k=1}^{m-1} a_k - \sum_{i=0}^{\infty} d_i\right] g \tag{9}$$

An alternative strategy for ensuring stationarity is to re-express the expectations themselves in terms of variables that are stationary by construction. As an example, the expectations can be formulated in terms of deviations from long run trends. This strategy is employed occasionally in FR-BDF, for example in the equation block used to determine corporate investment. See subsection 4.6.2 for details.

3.3 Computation of present values

This section describes the computation of present values under VAR-based and model-consistent expectations. We can divide expectations into two groups. Subsection 3.3.1 considers the first group. These expectations appear within a PAC equation such as (5). This computation is complicated by the fact that the discounting involved depends on the coefficients of the PAC polynomial.

The second group is considered in subsection 3.3.2. They contain present values that are used to directly compute the value of some model variables, e.g. the long run interest rate (see subsection 4.8.2 for details). They are not affected by any frictions, and can be constructed by a simple geometric discounting in the VAR-based case. In the MCE case, they resemble a recursive equation of a value function with a finite number of leads.

3.3.1 Present value of expected changes in the target in PAC equations

It is convenient for applications to decompose (5) into

$$\Delta y_t = a_0 \left(y^*_{t-1} - y_{t-1}\right) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \sum_{i=0}^{\infty} d_i \Delta \hat{y}^*_{t+i} + \sum_{i=0}^{\infty} d_i \Delta \bar{y}^*_{t+i} \tag{10}$$

where $\hat{y}^*_t$ and $\bar{y}^*_t$ are the stationary and nonstationary components of the target, i.e. $y^*_t = \hat{y}^*_t + \bar{y}^*_t$.[19] This is equivalent to

$$\Delta y_t = a_0 \left(y^*_{t-1} - y_{t-1}\right) + \sum_{k=1}^{m-1} a_k \Delta y_{t-k} + \mathrm{PV}\left(\Delta \hat{y}^*\right)_t + \mathrm{PV}\left(\Delta \bar{y}^*\right)_t \tag{11}$$

[18]This extended version of the standard PAC equation can also be derived from a modified version of the cost minimization problem given by (2) where changes in the decision variable would be centered by g, the exogenous growth rate of the target. An additional term $-b_0 \left(\Delta y_{t+i} - g\right)^2$ would appear within the sum, implying an additional term also within the PAC polynomial.

[19]These components are computed with a trend-cycle decomposition, e.g. an HP filter.

under the definitions

$$\mathrm{PV}\left(\Delta\hat{y}^{*}\right)_{t} \equiv \sum_{i=0}^{\infty} d_i \Delta\hat{y}^{*}_{t+i} \tag{12}$$

and

$$\mathrm{PV}\left(\Delta\bar{y}^{*}\right)_{t} \equiv \sum_{i=0}^{\infty} d_i \Delta\bar{y}^{*}_{t+i} \tag{13}$$

It is clear that these infinite sums cannot be used in applied work, and that equations in which these sums are replaced by e.g. recursive representations are needed. There are two different formulations – based on backward- and forward-looking expectations models – for how these finite expressions are constructed.

VAR-based expectations As in Section 3.1.1, the VAR on which the backward-looking version of the expectations model is based can be expressed in the form of (1):

$$Z_t = HZ_{t-1}$$

This equation can be iterated forward to find that for any i

$$Z_{t+i} = H^{i+1}Z_{t-1}$$

i.e. we can compute projections of the components of the VAR arbitrarily far into the future, including $\hat{y}^{*}_{t}$ and $\bar{y}^{*}_{t}$, if necessary. In practice, of course, the infinite sums of (12) and (13) remain impossible to operate with.

With some matrix algebra, it can be shown, however, that these infinite sums can be rearranged into finite expressions. Specifically, it can be shown that there are two $1 \times n$ vectors k_0 and k_1:

$$k_0 = A(1)A(\beta)\left[(\iota'_m I_m) \otimes H'\right]\left[I_{nm} - (G \otimes H')\right]^{-1}\left[\iota_m \otimes \iota_{k0}\right] \tag{14}$$

$$k_1 = A(1)A(\beta)\left[\left(\iota'_m \left(I_m - G\right)^{-1}\right) \otimes H'\right]\left[I_{nm} - (G \otimes H')\right]^{-1}\left[\iota_m \otimes \iota_{k1}\right] \tag{15}$$

where ι_{k0} and ι_{k1} are selection vectors for selecting the $k0$th and $k1$th elements of Z_t which correspond to $\hat{y}^{*}_{t}$ and $\bar{y}^{*}_{t}$. These vectors k_0 and k_1 can then be applied to compute finite representations of the expectations with the formulas

$$\mathrm{PV}\left(\Delta\hat{y}^{*}\right)_{t|t-1} = k_0 Z_{t-1} \tag{16}$$

and

$$\mathrm{PV}\left(\Delta\bar{y}^{*}\right)_{t|t-1} = k_1 Z_{t-1} \tag{17}$$

Note the subscript notation on the right hand side of equations (16) and (17). With the indices t and $t-1$ we wish to denote the fact that the computation of the discounting in the sum is done with respect to period t and that the future terms $\hat{y}^{*}_{t+i}$ and $\bar{y}^{*}_{t+i}$, $i > 0$ are constructed using the information set of period $t-1$.[20]

[20]This extra notation is unnecessary under MCE since, for agents with perfect foresight, the information set contains everything.

Model-consistent expectations The derivation of the finite expressions in the model-consistent case, with an information set ending in t, starts by multiplying (3) by $A(\beta F)^{-1}$ to obtain

$$A(L)y_t = A(\beta F)^{-1}A(\beta)A(1)\hat{y}_t^* + A(\beta F)^{-1}A(\beta)A(1)\bar{y}_t^* \tag{18}$$

which has exactly the same form as (5), with the exception that all terms involving y_t are now on the left hand side. Thus the first term on the right hand side must be equal to $\mathrm{PV}\left(\Delta\hat{y}^*\right)_t + A(1)\hat{y}_{t-1}^*$, and the second term to $\mathrm{PV}\left(\Delta\bar{y}^*\right)_t + A(1)\bar{y}_{t-1}^*$.

Thus for the first term we have (noting that $A(1) = a_0$)

$$\mathrm{PV}\left(\Delta\hat{y}^*\right)_t = -a_0\hat{y}_{t-1}^* + A(\beta F)^{-1}A(\beta)a_0\hat{y}_t^*$$

which is equivalent to

$$A(\beta F)\mathrm{PV}\left(\Delta\hat{y}^*\right)_t = a_0\left[A(\beta)\hat{y}_t^* - A(\beta F)\hat{y}_{t-1}^*\right]$$

resulting in

$$\mathrm{PV}\left(\Delta\hat{y}^*\right)_t = -\sum_{i=1}^{m}\alpha_i\beta^i\mathrm{PV}\left(\Delta\hat{y}^*\right)_{t+i} + a_0\left[\Delta\hat{y}_t^* + \sum_{k=1}^{m-1}\left(\sum_{j=k}^{m-1}\alpha_{j+1}\beta^{j+1}\right)\Delta\hat{y}_{t+k}^*\right] \tag{19}$$

Due to symmetry, an equivalent equation

$$\mathrm{PV}\left(\Delta\bar{y}^*\right)_t = -\sum_{i=1}^{m}\alpha_i\beta^i\mathrm{PV}\left(\Delta\bar{y}^*\right)_{t+i} + a_0\left[\Delta\bar{y}_t^* + \sum_{k=1}^{m-1}\left(\sum_{j=k}^{m-1}\alpha_{j+1}\beta^{j+1}\right)\Delta\bar{y}_{t+k}^*\right] \tag{20}$$

holds also for $\mathrm{PV}\left(\Delta\bar{y}^*\right)_t$, i.e. the trend component of expectations. These two equations – (19) and (20) – can be conveniently used in model-consistent applications instead of (16) and (17) that are used in VAR-based applications.

3.3.2 Present values with a constant discount factor

In some cases such as asset pricing equations, present values are discounted with a constant discount factor based on the expectation of the following sum:

$$(1-\beta)\sum_{i=0}^{\infty}\beta^i y_{t+i} \tag{21}$$

Under VAR-based expectations, with an information set ending in $t-1$, we compute the present value with the following formula:

$$\mathrm{PV}\left(y\right)_{t|t-1} = (1-\beta)\iota_h^{\prime}(I-\beta H)^{-1}HZ_{t-1}. \tag{22}$$

Under model-consistent expectations, with an information set ending in t, we compute the present value with the following recursive formula:

$$\mathrm{PV}\left(y\right)_t = \beta E_t\mathrm{PV}\left(y\right)_{t+1} + (1-\beta)y_t. \tag{23}$$

4 Model specification and estimation

This section describes the full model in detail. After describing the notation used, we proceed block by block, starting with supply and ending with public finances and the accounting framework. In what follows the equations that we present are written for the VAR-based case.

4.1 Estimation approach

As the model is estimated block by block, there is no need for a universal estimation methodology –the estimation methods of FR-BDF are highly dependent on the particular features of individual model components. However, given some interdependence between the various model components, there is some structure to the overall estimation procedure. The model was estimated using French Quarterly National Accounts in base 2014, from 1995-Q1 to 2017-Q4.[21] For the calculation of capital services (see section 4.3), we used wealth accounts in base 2010 and re-based capital and investment sub-components in 2014.

Many of the equations describing short run dynamics include expectation terms which require estimating a model to compute these terms. The model we apply is E-SAT. Given that E-SAT depends on the output gap, the estimation process is as follows: (i) we calibrate the production function consistently with estimated factor demand conditions and the factor-price frontier (see section 4.3) and obtain an estimate of the output gap, (ii) we estimate E-SAT using Bayesian methods and (iii) we estimate the rest of the model equations, using E-SAT to compute expectations in PAC equations.

The model equations are mostly estimated using a methodology that closely follows that of FRB/US, as the so called short run equations are all estimated with iterative OLS. The core equations of the backward-looking expectations model are estimated with Bayesian methods, while all auxiliary equations needed for expectations formation are estimated with OLS. All long run equations describing targets are estimated with simple OLS. Finally, some coefficients are calibrated, such as the production function parameters (except the labor/capital elasticity of substitution estimated as a parameter of the long run equation of business investment) and the markup of monopolistic firms.

The iterative estimation of the short run equations of the model is based on OLS: an initial guess is made on the PAC coefficients of the equation, which can be used to compute a discounted sequence of expectations using E-SAT, which is in turn used as an observable in the estimation of the PAC coefficients. Given these new estimates, the expectations sequence can be recomputed, the PAC coefficients re-estimated and so forth until convergence.

The estimation of equations following the PAC framework requires an assumption to be made regarding the discount factor β appearing in the cost minimization problem expressed in (2). We follow the approach of FRB/US and calibrate this number to be 0.98 in all main blocks of the model except household consumption (see section 4.6.1 for details). This discount factor is consistent with the real rate of return for financial assets of roughly 8%

[21] We used the 2018-Q1 QNA detailed figures published on 22 June 2018. Estimation samples can vary depending on: (i) equation specifications, (ii) data availability for variables that are external to QNA and (iii) modellers' judgment.

observed in the US during the postwar period (Brayton et al., 1996) and in France during the period 1970-2014, as shown by Garbinti et al. (2017).

It is important to note that the model is estimated with the VAR-based expectations for the reasons explained in the bird's-eye-view, see section 2.8. When FR-BDF is simulated with model-consistent expectations, it means that only the expectations are changed, but not the coefficients of the equations in which they enter. Take for instance the wage Phillips curve. The Phillips slope is estimated as an elasticity of wage inflation with respect to the expected discounted sum of future unemployment gaps. The latter is the policy function of the E-SAT variables in $t-1$. In the MCE case, expectations take on a forward-looking form but the slope remains unchanged, because we assume that on a "historical" sample these expected sums are the same in both cases.

4.2 Notation

While the model notation in code follows very closely the standard notation laid out in the System of National Accounts (United Nations, 2009), we prefer here to use shorter notations in order to present more compact formulas. In addition, given that FR-BDF is a very large model that e.g. makes strong distinctions between agents and sectors and contains explicitly prices, volumes and values for the same economic concepts while using certain complex mathematical transformations, it is necessarily complexity and somewhat opaque. In this subsection we attempt to clarify these issues.

Our notation introduces a number of operators that are particularly common in FR-BDF. The first is the expectations operator, which we denote by $\mathrm{PV}\left(x\right)_{t|t-k}$ where x_t is a model variable. The subscript describes the timing of information: the first component t refers to the date when the expectation is constructed, while the second component $t-k$ refers to the information set available for the construction. The second is the gap, by which we mean the deviation of x_t from its long run trend $\bar{x}_t$. This gap is denoted by $\hat{x}_t$, i.e. $\hat{x}_t = x_t - \bar{x}_t$.

Furthermore, we follow some typographical conventions in order to simplify our notation. Lower-case letters will be used to denote logarithms – e.g. the logarithm of household consumption C_t will be c_t – or interest rates, e.g. the short rate will be denoted i_t and the weighted average cost of capital $wacc_t$. Other rates or ratios will be denoted with the letter τ.

Finally, Table 4.2.1 presents, for the sake of convenience, the variables and notation for the core variables appearing in the expectations satellite model E-SAT, which is transverse across FR-BDF.

4.3 Supply block

FR-BDF has a fully specified supply block which enables the model to endogenously converge toward the long run or natural level of GDP (see section 5.1), as well as to define the output gap used in E-SAT. The model is based on a standard neoclassical growth model to which we add monopolistic competition.

We model separately the *market branches* and the *non-market branches*.[22] Market

[22] Market branches group agriculture (AZ in Insee codification), industry (DE and C codes), construction

Table 4.2.1: E-SAT core variables

Notation	Description
$\hat{y}_t$	Output gap, in deviation from the long run output
i_t	Short run interest rate, 3-month Euribor
$\bar{i}_t$	Long run trend of the short run interest rate
$\pi_{Q,t}$	Value added price inflation of market branches
$\bar{\pi}_{Q,t}$	Long run trend of the value added price inflation of market branches
$\hat{y}_{EA,t}$	Euro area output gap, in deviation from potential output
$\pi_{EA,t}$	Growth rate of the euro area GDP deflator
$\bar{\pi}_{EA,t}$	Long run trend of the GDP deflator inflation

branches' supply is based on a micro-founded theoretical framework with a production function technology and equilibrium conditions derived in absence of adjustment costs. In contrast, for non-market branches output, we do not assume any production function technology. This is furthermore justified by the fact that these variables are basically exogenous in conditional forecasting; in unconditional simulation, they are assumed to grow at the same rate as long-run real GDP.

This section presents the theoretical framework from which we derive the specification of desired targets for business investment, employment and value added price.[23] We then present the "model-consistent" calibration of FR-BDF's production function of firms. Finally, we define the level of long run output toward which FR-BDF converges endogenously.

4.3.1 Theoretical framework of market branches' supply

At the level of market branches, the production function technology is assumed to be a Constant Elasticity of Substitution (CES) production function, rather than a Cobb-Douglas production function. This makes it possible to have a more plausible non-unitary elasticity of labor and investment to labor cost and capital user cost. In addition, we are able to preserve the Balanced-Growth Path hypothesis by assuming a labor-augmenting technical progress. In this subsection, we derive equilibrium conditions for capital services, employment and value added price in the absence of adjustment costs.

Production function technology Market branches' aggregate value added Q_t is determined by the following CES production function

$$Q_t = F(K_t, H_t N_t, E_t) \equiv \gamma \left[\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(E_t H_t N_t\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{24}$$

(FZ code) and services (codes going from GZ to MN). Non-market branches correspond to the branch of non-market services (OQ code).

[23] The estimation of corresponding short run equations and the properties of these blocks will be detailed in sections 4.6.2, 4.5.2 and 4.4.

Table 4.3.1: Variables used in Section 4.3

Notation	Description
Q_t	Value added of market branches, volume
$P_{Q,t}$	Value added deflator of market branches
K_t	Capital services, market branches excluding agricultural and real estate branches, volume
$\tilde{I}_t$	Investment, market branches excluding agricultural and real estate branches, volume
δ_t	Depreciation rate of capital
N_t	Total employment of market branches, thousands of persons
$N_{S,t}$	Salaried employment of market branches, thousands of persons
H_t	Working time per capita in market branches, hours
E_t	Solow Residual of market branches
$\bar{E}_t$	Trend labor efficiency (labor-augmenting technological progress)
$\tilde{W}_t$	Total labor cost per worker, value, market branches
$\tilde{r}_{K,t}$	Real user cost of capital, excluding agricultural, real estate and public sector
$wacc_t$	Weighted average cost of capital
π_Q	Value added price inflation at time t conditional on information at time $t-1$, using E-SAT
$Q'_{K,t}$	Marginal product or return on capital, volume
$u_{N,t}$	Long run equilibrium level of unemployment, in percentage
$\bar{\psi}_t$	HP-trend share of market branches employment in total employment
$\overline{Pop}_t$	HP-trend of labor force, in thousands of persons
Q_t^{nm}	Non-market branches GDP, volume
Y_t	Total economy GDP, volume
$u_{N,t}$	Long run equilibrium level of unemployment, in percentage
$Q_{N,t}$	Market branches' long run value added, volume
$Y_{N,t}$	Total economy long run GDP, volume

Note: tilde-marked variables are specific to this section. We use it to make a distinction between total business investment of market branches I_t and total business investment excluding agricultural, real estate and public branches investment $\tilde{I}_t$; the same applies to real user cost of capital services $\tilde{r}_{K,t}$. $\tilde{W}_t$ denotes total labor cost, i.e. gross wages *plus* employers' social contributions.

where K_t is a measure of capital services, $N_t H_t$ denotes labor supply measured in total hours and E_t a labor-augmenting technical progress;σ is the elasticity of substitution between labor and capital.[24] E_t is obtained as a Solow residual by inverting the production function (24) such that:

$$E_t = \left[\frac{\left(\frac{Q_t}{\gamma}\right)^{\frac{\sigma-1}{\sigma}} - \alpha K_t^{\frac{\sigma-1}{\sigma}}}{(1-\alpha)(H_t N_t)^{\frac{\sigma-1}{\sigma}}} \right]^{\frac{\sigma}{\sigma-1}} \tag{25}$$

Input demand and factor price frontier Under monopolistic competition, firms maximize profit by setting a constant optimal markup μ. Profit maximization yields the following standard first order conditions, determining the long run equilibrium levels of capital and labor demand, which equate marginal productivity and markup-augmented marginal cost for each factor:

$$\frac{\tilde{r}_{K,t}}{P_{Q,t}} = \frac{\alpha}{\mu} \gamma^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t}{K_t} \right)^{\frac{1}{\sigma}} \tag{26}$$

$$\frac{\tilde{W}_t}{P_{Q,t}} = \frac{1-\alpha}{\mu} \gamma^{\frac{\sigma-1}{\sigma}} E_t H_t \left(\frac{Q_t}{E_t H_t N_t} \right)^{\frac{1}{\sigma}} \tag{27}$$

The real user cost of capital services is defined as:

$$\frac{\tilde{r}_{K,t}}{P_{Q,t}} = \left(wacc_t + \tilde{\delta}_t - PV(\pi_Q)_{t|t-1} \right) \frac{P_{\tilde{I},t}}{P_{Q,t}} \tag{28}$$

where $PV(\pi_Q)_{t|t-1}$ is the expected present-value of VA price inflation at time t conditional on information at time $t-1$ obtained from E-SAT, $wacc_t$ is the weighted average cost of capital and $P_{\tilde{I},t}/P_{Q,t}$ is the relative price of business investment to VA price. Our measure of the real user cost of capital implicitly assumes no adjustment costs; see section 4.6.2 for more details.

Hence, using production function (24) and labor demand (27), we derive the following Factor Price Frontier (FPF, henceforth):

$$P_{Q,t} = \frac{\mu}{\gamma} (1-\alpha)^{\frac{\sigma}{1-\sigma}} \left[1 - \alpha^{\sigma} \left(\frac{Q'_{K,t}}{\gamma} \right)^{1-\sigma} \right]^{-\frac{1}{1-\sigma}} \frac{\tilde{W}_t}{E_t H_t} \tag{29}$$

where marginal productivity or return on capital is defined by the partial derivative of production function (24) with respect to K_t

$$Q'_{K,t} \equiv \alpha \left(\gamma \right)^{\frac{\sigma-1}{\sigma}} \left(\frac{Q_t}{K_t} \right)^{\frac{1}{\sigma}} \tag{30}$$

This non-standard version of the FPF calls for some comments. In structural macroeconomic models, the standard FPF generally relates the output price to the labor cost and

[24] In the measurement of K_t, we exclude agricultural (AZ code), housing services (LZ code) and non-market services (OQ code) capital to focus on productive capital. Capital services are computed using national wealth accounts with a methodology close to Cabannes et al. (2013).

the user cost of capital. Such a relation is usually derived using both capital and labor demand conditions, i.e. assuming both capital and labor markets are at equilibrium. Afterwards, because of adjustment costs on investment in FR-BDF (see section 3.2), capital will be inelastic, while the real cost of capital $\tilde{R}_{K,t}$ is computed under the assumption of the absence of adjustment costs (i.e. expected capital gains are assumed to be equal to expected inflation, see section 4.6.2). Hence, we do not rely on condition (26) that the capital market is at equilibrium when deriving the FPF. As a result, firms' pricing behavior depends on the marginal return on capital rather than on the real user cost of capital. We interpret this specification of the FPF as a resource constraint with the labor market at its long run equilibrium (27).

4.3.2 Equilibrium of the supply of market branches

Specification FR-BDF's supply block is derived from equations (27), (26) and (29) which act as a resource constraint. We now distinguish equilibrium variables (K_t^*, N_t^*, P_t^*) from actual variables (K_t, N_t, P_t) denoting the first ones with stars. These equilibrium variables define the targets of behavioral short run equations, which take into account the presence of adjustment costs (presented in the following sections). In practice, we slightly deviate in some cases from the theoretical equations derived earlier, in order to provide a better fit to the data.

First, following FRB/US, we transform the capital demand condition (26) into an investment demand equation. In order to do so, we define the equilibrium capital K_t^* by

$$K_t^* \equiv \left(\frac{\alpha}{\mu}\right)^{\sigma} \gamma^{\sigma-1} \left(\frac{\tilde{r}_{K,t}}{P_{Q,t}}\right)^{-\sigma} Q_t \tag{31}$$

Then, we evaluate the equilibrium investment from the steady-state of the capital accumulation equation:

$$K_t = (1 - \tilde{\delta}_t)K_{t-1} + \tilde{I}_t \tag{32}$$

where $\tilde{\delta}$ is the depreciation rate of capital and $\tilde{I}_t$ is the investment of market branches, excluding agricultural and housing services. We model capital services with such an aggregate capital accumulation formula for the sake of simplicity, although capital accumulation occurs in national accounts at the level of each asset.[25] Let denote the trend growth rate of capital by g_t^K measured by the HP filter and rewrite (32) to get the definition of equilibrium investment:

$$\tilde{I}_t^* \equiv \frac{\tilde{\delta}_t + g_t^K}{1 + g_t^K} K_t^* \tag{33}$$

Then, replacing K_t^* by (33) in (31) yields an optimal demand for aggregate market branches investment $\tilde{I}_t^*$:

$$\tilde{I}_t^* \equiv \frac{\tilde{\delta}_t + g_t^K}{1 + g_t^K} \left(\frac{\alpha}{\mu}\right)^{\sigma} \gamma^{\sigma-1} \left(\frac{\tilde{r}_{K,t}}{P_{Q,t}}\right)^{-\sigma} Q_t \tag{34}$$

[25] In our computation of capital services, for each asset, capital services are assumed to be proportional to the net capital stock of this asset, which has in national accounts its specific depreciation rate.

Finally, the equilibrium investment target is directly derived from (34) by taking logs:

$$\log \tilde{I}_t^* = a_0 + \log(Q_t) - \sigma \log\left(\frac{\tilde{r}_{K,t}}{P_{Q,t}}\right) + \log\left(\frac{\tilde{\delta}_t + g_t^K}{1 + g_t^K}\right) \tag{35}$$

Second, the Solow residual E_t is replaced by trend labor efficiency $\bar{E}_t$ in each equation. This choice is motivated by the fact that the Solow residual is volatile and combines both cyclical and structural factors, which are unrelated to the labor-specific technical progress. In particular, the Solow residual does not take account of the utilization rate of production capacity. As a result, we assume equilibrium demand conditions for labor and investment as well as the evaluation of FPF at the trend level of labor efficiency $\bar{E}_t$. Trend efficiency is defined as the level of efficiency with a utilization rate of production capacity equal to its long run mean and estimated with the following three assumptions: (i) $\bar{E}_t$ follows a deterministic trend with multiple breaks in its slope, (ii) there is a break in level (or "step") in 2008-Q3 to account for a permanent effect of the 2008-09 recession and (iii) there is an autoregressive structure to account for a smooth transmission of shocks (e.g. the 2008-Q3 step). Figure 4.3.1 represents both the Solow residual and the estimated trend labor efficiency whose annual growth rate is estimated at 2.4% before 2002-Q2 and at 0.85% afterwards. The 2008-Q3 step is estimated at 4.2%.

Third, we choose to estimate a labor-demand equation for *salaried* employment instead of *total* employment of market branches[26]. This choice is mostly due to a better econometric performance of the salaried employment equation (see section 4.5.2 for details). However, in order to bridge our theoretical model with the estimated equation for labor demand, we would need to recover the estimates of the total employment equation. Taking logs of (27), we obtain:

$$\log(N_t^*) = \tilde{b}_0 + \log(Q_t) - \log(\bar{E}_t) - \sigma \log \frac{\tilde{W}}{P_{Q,t}\bar{E}_t} + (\sigma - 1)\log(H_t) \tag{36}$$

Then define $\nu_t = N_{S,t}/N_t$. Note that the OLS estimate of $\tilde{b}_0$ is equal to $b_0 - \overline{\log(\nu)}$ where $\overline{\log(\nu)}$ is the empirical mean of $\log \nu_t$ and b_0 is the estimated intercept of the following *salaried* employment equation:

$$\log N_{S,t}^* = b_0 + \log(Q_t) - \log(\bar{E}_t) - \sigma \log \frac{\tilde{W}}{P_{Q,t}\bar{E}_t} + (\sigma - 1)\log(H_t) \tag{37}$$

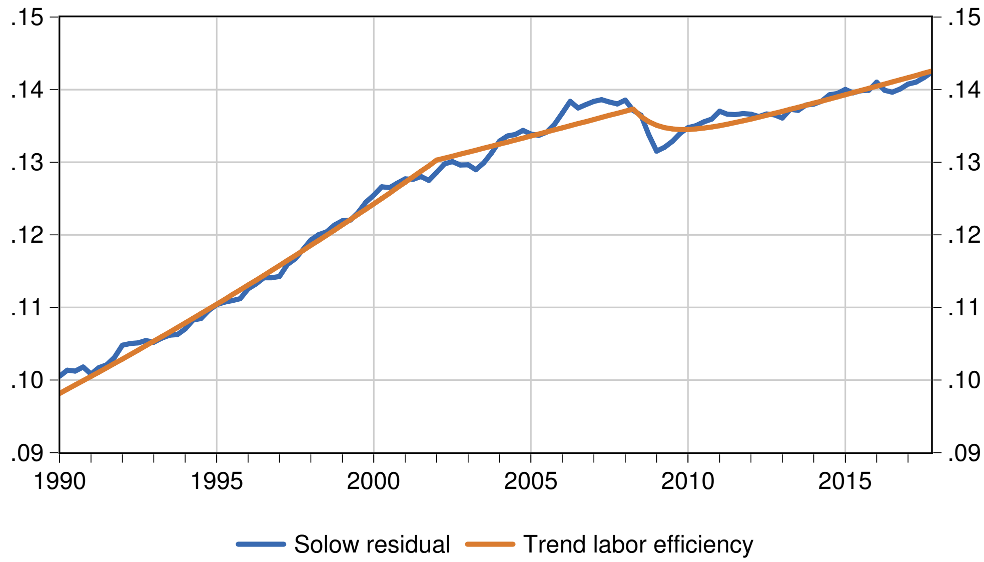
As a result, we can recover the estimate for $\tilde{b}_0$ from the intercept b_0 and from $\overline{\log(\nu)}$ still without directly estimating equation (36).[27]

Fourth, we use a measure of trend return on capital $\overline{Q'_{K,t}}$ rather than observed return $Q'_{K,t}$. The trend is obtained by using an HP filter on observed $Q'_{K,t}$ in the historical sample but is projected according to an exogenous AR(1) process, anchored to the steady-state of

[26] Indeed, equation (27) defines the demand for *total* employment. Within our sample (1995Q1-2017Q4), the evolution of the share of salaried employment in market branches has been small: it rose from around 86% in 1995Q1 to a peak around 89% in the mid-2000s and returned to 88% in 2017Q4.

[27] This is notably due to the fact that the only estimated parameter is the intercept, while σ is calibrated.

Figure 4.3.1: Solow residual and trend labor efficiency



equation (26) (see section 4.9). Hence, for given values of σ and α the estimated equation for the target of VA price equilibrium is:

$$\log(P^*_{Q,t}) = c_0 + \frac{\sigma}{1-\sigma}\log(1-\alpha) - \frac{1}{1-\sigma}\log\left[1-\alpha^{\sigma}\left(\frac{\overline{Q'_{K,t}}}{\gamma}\right)^{1-\sigma}\right] + \log\frac{\tilde{W}_t}{\bar{E}_t H_t} \tag{38}$$

Figure 4.3.2 shows the marginal return of capital $Q'_{K,t}$ and the trend marginal return of capital $\overline{Q'_{K,t}}$, both annualized. First, the annual marginal return of capital fluctuates between 30% and 22%. This value for $Q'_{K,t}$ is broadly in line with the level of the real user cost of capital augmented by the markup.[28] Second, we observe a downward trend in the marginal return of capital because capital services have grown faster than real output since the early 2000s.[29]

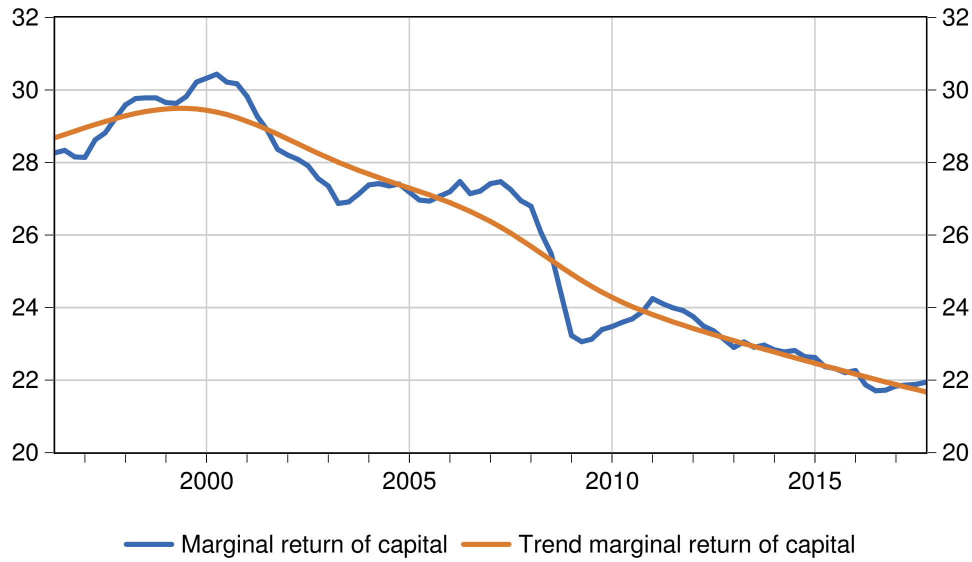
Calibration We now turn to the calibration procedure of the production function. We label it "model-consistent calibration" since the production function's parameters (α, σ and γ), markup (μ) and trend labor efficiency are calibrated consistently with estimated equilibrium conditions (35), (37) and (38).[30] Indeed, the calibration of the production function, the

[28]The markup μ is calibrated at 1.31. The annualized real user cost of capital fluctuates between 21% and 18%. Regarding its subcomponents, the WACC fluctuates between 8% and 5% (see Figure 4.8.2), the depreciation rate is equal to 15% per year on average between 1995 and 2017 and expected inflation is around 2% (annual rate).

[29]Marginal return of capital $Q'_{K,t}$ is homogenous to the output-to-capital ratio Q_t/K_t, see eq. (30) and therefore its growth depends on the relative growth of real output and capital services.

[30]Contrary to other authors such as Klump et al. (2012), we neither normalize the CES production function nor calibrate it with big ratios (i.e. capital share, labor share, capital/output ratio), because the resulting

Figure 4.3.2: Observed $Q'_{K,t}$ and trend $\overline{Q'_{K,t}}$ annual marginal return of capital, in percentage



estimation of labor efficiency and estimates of equilibrium equations for business investment, employment and VA price are mutually dependent.

From equation (25), the calibration of the production function's parameters (α, σ and γ) determines the level of both the Solow residual and trend labor efficiency, which determines target labor demand and VA price equations. In turn, intercepts of the three equilibrium equations – investment and labor demand and VA price – are non-linear functions of these parameters and of the markup μ. Let (a_0, b_0, c_0) be respectively the three estimated intercepts of equations (35), (37) and (38). If the production function and markup were estimated consistently with these equations, we should meet the following cross-restrictions between estimated and theoretical intercepts:

$$a_0 = \log\left[\left(\frac{\alpha}{\mu}\right)^{\sigma}(\gamma)^{\sigma-1}\right] \tag{39}$$

$$b_0 = \log\left[\left(\frac{1-\alpha}{\mu}\right)^{\sigma}(\gamma)^{\sigma-1}\right] + \overline{\log\nu} \tag{40}$$

$$c_0 = \log\left[\frac{\mu}{\gamma}\right] \tag{41}$$

where the theoretical intercepts are the right-hand side terms. Since FR-BDF is estimated equation-by-equation, applying cross-restrictions without a joint-estimation of these equations is challenging. Hence, we developed a strategy to estimate FR-BDF sequentially, without departing from the equation-by-equation estimation strategy.

calibration would not necessarily be consistent with estimated behavioral equations, where cross-restrictions would not have been imposed.

As a first step,we obtain an estimate for the capital-labor elasticity of substitution $\sigma = 0.53$ from the firms' *target* investment I_t^* equation(63), which is independent of the trend efficiency estimate and has quite a similar structure to equation (35), albeit with some differences (see section 4.6.2 for more details). In this step, we specifically assume $PV(\pi_Q)_{t|t-1} = \bar{\pi}_t$ and evaluate $r_{K,t}$ in equation (63) at the long run anchor of inflation expectations, because E-SAT has not been estimated at this stage. We do not estimate σ using $\tilde{I}_t$ since it is not available from quarterly national accounts and for which we construct a quarterly measure from annual data by interpolation.

As a second step, we obtain an estimate of the investment equation's intercept a_0. At first glance, we have a system of three equations (39), (40), (41) and three unknowns (α, γ, μ), which we would solve by grid search. However, it can be reduced to a two equation–two unknown system since business investment equation (35) does not depend on trend efficiency. Consequently, the investment equation's intercept a_0 can be estimated outside the grid search and we can deduce markup μ_i as a function of estimated a_0, calibrated σ and grid parameters (α_i, γ_i) from the following equation:

$$\begin{aligned}\mu_i &= \mu(\alpha_i, \gamma_i, \sigma, a_0) \\ &= \exp\left(\log \alpha_i + \frac{\sigma - 1}{\sigma} \log \gamma_i - \frac{a_0}{\sigma}\right)\end{aligned} \tag{42}$$

Similarly to $r_{K,t}$ in the first step, we evaluate $\tilde{r}_{K,t}$ at the long run anchor of inflation expectations for the reasons exposed above.

As a third step, we use a grid search to obtain the calibration of the FR-BDF production function that is consistent with estimated intercepts, with σ being calibrated. We run the grid search for plausible but sufficiently large ranges of α and γ which we expect to be in $[0.2, 0.4]$, with a step parameter equal to 0.001. Consequently, we have exactly 201 points for each parameter, which results in 40401 grid points. Hence, for each point (α_i, γ_i) of the grid $(\alpha, \gamma)^2$, the grid search proceeds as follows:

1. Calculate μ_i and $Q'_{K,t}$ using equation (30).[31]

2. Compute the Solow Residual E_t from equation (25) and estimate trend labor efficiency $\bar{E}_t$ with the method described above.

3. Estimate b_0 and c_0 by OLS from equations (37) and (38), for a given $\bar{E}_t$ and for parameters $(\sigma, \alpha_i, \gamma_i, \mu_i)$.

4. Define the following $\ell 1$-norm:

$$||\mathbf{x}||_1 := \left| b_0 - \log\left(\frac{1-\alpha_i}{\mu_i}\right)^{\sigma} \left(\gamma_i\right)^{\sigma-1} - \overline{\log \nu} \right| + \left| c_0 - \log \frac{\mu_i}{\gamma_i} \right|$$

[31]In practice, we do not estimate the VA price equation using the HP-trend return on capital $\overline{Q'_{K,t}}$ but with $Q'_{K,t}$. Since the only estimated parameter is the intercept, the OLS estimator of c_0 only depends on regressors' *empirical means*. Thus, using the observed or HP-trend return on capital is perfectly equivalent because they have the same historical mean.

5. Finally, pick the grid point which minimizes $||\mathbf{x}||_1$:

$$(\alpha, \gamma) = \underset{\alpha_i, \gamma_i}{\arg\min} ||\mathbf{x}||_1$$

provided $\min||\mathbf{x}||_1 < 10^{-3}$ to obtain μ according to equation (42).

Table 4.3.2 summarizes the model-consistent calibration we obtain and compares it to what we would obtain with the normalization of the CES and big ratios. The calibration that we obtain is consistent with estimated equilibrium conditions (business investment, employment and VA price) and differs slightly from the big ratios method, especially for markup μ and scale parameter γ.

Table 4.3.2: Calibration of the FR-BDF production function

	σ	α	γ	μ	$\min\|\|\mathbf{x}\|\|_1$
Model-consistent calibration	0.53	0.26	0.34	1.31	0.0006
Calibration using big ratios	0.53	0.26	0.29	1.42	-

The model-consistent calibration is critical for the long run convergence toward the long run levels of GDP Y_N and sector value added Q_N (see section 4.3.3). In contrast, if we were calibrating the production function using the method of big ratios, the only way for the VA to converge toward its long run level would be to calibrate the long run equilibrium equations' intercepts (a_0, b_0, c_0) to their theoretical values. Such a strategy would inevitably worsen the empirical fit of estimated equations. In addition, normalization and big ratios would require that the economy be on a balanced growth path in the historical sample – which is not verified, in particular regarding the capital/output ratio.

4.3.3 Long run output in FR-BDF

We now present the long run output of FR-BDF. As in the traditional approach of potential output, we define here long run output as the level of output that can be achieved for a given level of capital services. We use a specific name for this object, because some assumptions have been made specifically to be consistent with the long run of FR-BDF, while the Banque de France estimates of potential output do not necessarily adopt these constraints.

In contrast, the concept of long run output in FR-BDF differs from the one usually found in DSGE models. In FR-BDF, rigidities, whether real or nominal, are captured by the costs of adjusting variables toward their targets. The long run output is defined by the state of the economy in which both nominal and real rigidities have waned, except for capital services considered as given. On the contrary, in DSGE models, long run (or natural) output is generally defined as the flexible-price level of output, i.e. the level of output in the absence of nominal rigidities but given real rigidities.

As mentioned earlier, market and non-market branches are treated differently and separately. The long run output of market branches is derived from a production function, while non-market branches' long run output is defined by a statistical trend. Finally, the long run output of the total economy is calculated using a chained price aggregation of these two sub-components.

Long run output of market branches The long run output of market branches is derived from the production function given exogenous trend of the labor force, trend efficiency and given actual level of capital services.[32] Hence, the long run level of sector value added is defined by:

$$Q_{N,t} = \gamma \left[\alpha K_t^{\frac{\sigma-1}{\sigma}} + (1-\alpha) \left(\bar{E}_t \bar{H}_t \bar{\psi}_t (1-u_{N,t}) \overline{Pop_t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{43}$$

where $\bar{H}_t$ denotes the HP-trend hours per worker. Since long run equilibrium unemployment $u_{N,t}$ is estimated on the total economy employment instead of total employment, we need to define $\bar{\psi}_t$ as the trend share of market employment in total employment, in order to recover the trend of total employment. As a result, $\bar{H}_t \bar{\psi}_t (1-u_{N,t}) \overline{Pop_t}$ represents trend of labor supply (in hours).

Besides trend efficiency $\bar{E}_t$ (see above) and long run unemployment $u_{N,t}$ which have specific estimates, both trend hours and trend share of employment in total employment are obtained from a standard HP filter with $\lambda = 1600$ on quarterly data.[33] The long run unemployment rate is estimated outside FR-BDF by Kalman filtering, as a time-varying intercept of a price-inflation Phillips curve.[34] Finally, we define the market branches' output gap by $\hat{Q}_t \equiv Q_t / Q_{N,t} - 1$.

Long run output of non-market branches First, we define the output non-market branches Y_t^{nm} as the chained price difference between GDP (Y_t) and market branches' value added (Q_t), which includes the value-added of public and non-profit institutions serving households (NPISH) sectors but also taxes and subventions on products. Hence, the long run output of non-market branches $\bar{Y}_t^{nm}$ is defined by a statistical trend and is projected such that the non-market output gap will close mechanically, like other trend variables in FR-BDF (see sections 4.9). In practice, we measure the non-market branches' trend GDP using the HP filter with a smoothing parameter $\lambda = 1600$. In future developments of FR-BDF, we could introduce an explicit production function technology for the non-market branches, which is mostly the public sector.

Aggregation and total economy long run output Finally, the long run output of the total economy $Y_{N,t}$ is obtained by chained-prices aggregation of $Q_{N,t}$ and $\bar{Q}_t^{nm}$ and the output gap of the total economy is denoted by $\hat{Y}_t \equiv Y_t / Y_{N,t} - 1$. At the balanced-growth path, the long-run output of total economy $Y_{N,t}$ as well as its sub-components $Q_{N,t}$ and $\bar{Q}_t^{nm}$ grow at a constant annual rate $\Delta \bar{y}_N$, which is the sum of the growth rate of labor efficiency $\Delta \bar{e}$ and the growth rate of labor force $\Delta \bar{\ell}$. We estimate the trend annual growth-rate of labor efficiency at $\Delta \bar{e} = 0.85\%$, see section 4.3. Regarding labor force, we assume a trend annual growth-rate equal to $\Delta \bar{\ell} = 0.2\%$, which is the average growth-rate of labor force over the

[32] We use the actual level of capital services because we assume that capital does not deviate much from its long run level.

[33] The trend share of market employment is quite stable over the sample: it increases from around 69.8% in 1996Q1 to 71.2% in 2002Q4 and returns to 70.2% in 2017Q4.

[34] This estimate is provided by the external trade and structural policies studies unit of Banque de France. Another approach left for further research could consist in calculating an equilibrium unemployment rate directly from the wage Phillips curve of FR-BDF.

period 2015-2045 based on Insee labor force projections for 2070 published in 2017.[35] Given these technical assumptions, the long-run output growth-rate $\Delta\bar{y}_N$ is equal to 1.05% at the balanced-growth path.

4.4 Value added price of market branches

The value added price equation is one of the key equation in FR-BDF since this deflator enters the equations of all the other types of prices. It enables expectations to affect price setting.

Table 4.4.1: Variables used in section 4.4

Notation	Description
$\overline{Q'_{K,t}}$	Hodrick-Prescott filter of marginal product or return on capital, volume
H_t	Working time per capita (in hours)
$\tilde{W}_t$	Total labor cost per capita, gross wages plus employers' social contributions, value
$\bar{E}_t$	Long run efficiency
$\Delta\bar{e}_t$	Efficiency trend, $\Delta log(\bar{E}_t)$
$p_{Q,t}$	Value added price of market branch (in log)
$p^*_{Q,t}$	Target of the value added price (in log)
$\bar{\pi}^*_{Q,t}$	Hodrick-Prescott filter of $\pi^*_{Q,t}$
$\bar{\pi}_t$	Long run anchor of inflation
$\pi_{Q,t}$	Value added price inflation
$\hat{y}_t$	Output gap
$\pi_{W,t}$	Growth rate of wage per capita of market branches
$PV(\pi^*_Q)_t$	Present value of the expected growth rate of the value added price target
$\bar{\pi}$	Long run value of inflation
L	Lag operator
E-SAT	See Table 4.2.1
$\tilde{\pi}_{W,t}$	Efficient wage inflation
$\hat{u}_t$	Unemployment gap

Target The long run of the VA price is derived from the price frontier. It is given by equation (38) described in detail in section 4.3.1, where the price target depends on the marginal productivity of capital (instead of its user cost) and on the efficient hourly labor

[35]Insee projections are available at: `https://www.insee.fr/fr/statistiques/2844302`.

cost. For the sake of convenience we provide it below:

$$p^*_{Q,t} = c_0 + \frac{\sigma}{1-\sigma}\log(1-\alpha) - \frac{1}{1-\sigma}\log\left[1-\alpha^\sigma\left(\frac{\overline{Q'_{K,t}}}{\gamma}\right)^{1-\sigma}\right] + \log\frac{\tilde{W}_t}{\bar{E}_t H_t}$$

All parameters except the constant are calibrated or estimated within other equations, see Table 4.4.2.

Table 4.4.2: Estimates and calibrated parameters, value added price, long run

	Coef.	s.e.
σ	0.53	-
α	0.26	-
μ	0.335	-
γ	1.31	0.001
$R^2 = 0.97$		

Short run equation The short run is estimated using the PAC framework on the 1997Q1-2017Q4 sample. We added a direct effect of current demand in the short run which is supposed to capture in a reduced form the behavior of non-optimizing firms. The parameter estimates are presented in Table 4.4.3.

$$\begin{aligned}\pi_{Q,t} = PV(\pi^*_Q)_{t|t-1} + \beta_0\left[p^*_{Q,t-1} - p_{Q,t-1}\right] + \beta_1\pi_{Q,t-1} + \beta_2\hat{y}_t \\ +(1-\beta_1-\omega)\bar{\pi}^*_{Q,t-1} + \epsilon_t\end{aligned} \tag{44}$$

Table 4.4.3: Estimates and calibrated parameters, value added price, short run

	Coef.	s.e.
β_0	0.06	0.02
β_1	0.50	0.09
β_2	0.09	0.03
ω	0.46	-
$R^2 = 0.40$		

Expectations In order to build expectations of the growth rate of the value added price target we had to include three additional equations into the core of the E-SAT model. The first one is to describe the change in the target itself. Since we are constrained in the number of auxiliary equations that we can add, the easiest way to model the growth rate of the value added price target is to link it to the real efficient wage, which was actually used to construct this target[36]:

$$\pi^*_{Q,t} = \beta_0(\pi_{W,t} - \Delta\bar{e}_t) + (1-\beta_0)\bar{\pi}^*_{Q,t} + \epsilon_t \tag{45}$$

[36]Note however that here we are using net wage rather than gross wage as in the target equation.

It would be logical to use $\bar{\pi}_t$ as a trend instead of $\bar{\pi}^*_{Q,t}$ but then we would need a constant to center the residuals.[37] The second equation is a Phillips curve, linking the real effective wage to the unemployment gap:

$$(1 - \rho L)\left[\pi_{w,t} - \Delta\bar{e}_t - \bar{\pi}^*_{Q,t}\right] = \beta_0 \hat{u}_t + \epsilon_t \tag{46}$$

where L is a lag operator.

The third equation mimics an Okun's law, relating the unemployment gap to the output gap with an AR(1) process in residuals:

$$\begin{cases} \hat{u}_t = \beta_0 \hat{y}_t + \eta t \\ \eta_t = \rho \eta_{t-1} + \epsilon_t \end{cases} \Rightarrow \hat{u}_t = \beta_0(\hat{y}_t - \rho \hat{y}_{t-1}) + \rho \hat{u}_{t-1} + \epsilon_t \tag{47}$$

The estimated parameters are available in Table 4.4.4. As was already mentioned, we did not find a high R^2 of auxiliary equations, but for meaningful coefficients. β_0 from the Okun's law in equation (47) confirms our prior, see discussion in subsection 4.5.1. A negative coefficient in front of $\hat{y}_{t-1}$ in the policy function is offset by a positive coefficient with respect to $\hat{u}_{t-1}$. The elasticity of the value added price with respect to the output gap is, in the end positive: $\frac{\pi_{Q,t}}{\hat{y}_{t-1}} = (-0.15 + 1.1 \cdot 0.246) \cdot 10^{-2} = 1.3 \cdot 10^{-3} > 0$. The slope of the Phillips curve measured by the ratio $\frac{\beta_0}{1-\rho}$ from equation (46), is equal to 0.53, which is a bit higher than the one computed from a partial equilibrium of the wage Phillips curve (see equation (52)) but still lies in the boundaries of commonly accepted values. A high elasticity of $PV(\pi^*_Q)_{t|t-1}$ with respect to $\bar{\pi}^*_{Q,t}$ in the policy function is related to a constraint of the balanced growth path condition of equation (45). However, it has no implication for the forecasting exercise or policy experiments since $\bar{\pi}^*_{Q,t}$ is exogenous in the model:

$$\bar{\pi}^*_{Q,t} = \alpha_0 \bar{\pi}^*_{Q,t-1} + (1 - \alpha_0)\bar{\pi} \tag{48}$$

where α_0 is calibrated to 0.95 (which corresponds to a half life of 12 quarters) and $\bar{\pi}$ is 0.0048.

Dynamic contributions Figure 4.4.1 describes how the various terms of the long and short run equations of the value added price have contributed to variation in its growth rate. Almost all positive dynamics of the latter are explained by the value added inflation target. Expectations are as important for the dynamics of the value added price inflation as for the output gap. After the second half of 2009, the expectations put a downward pressure on the value added inflation. Indeed, these expectations are strongly influenced by the trend inflation of the price target, which is subdued in the post-crisis period.

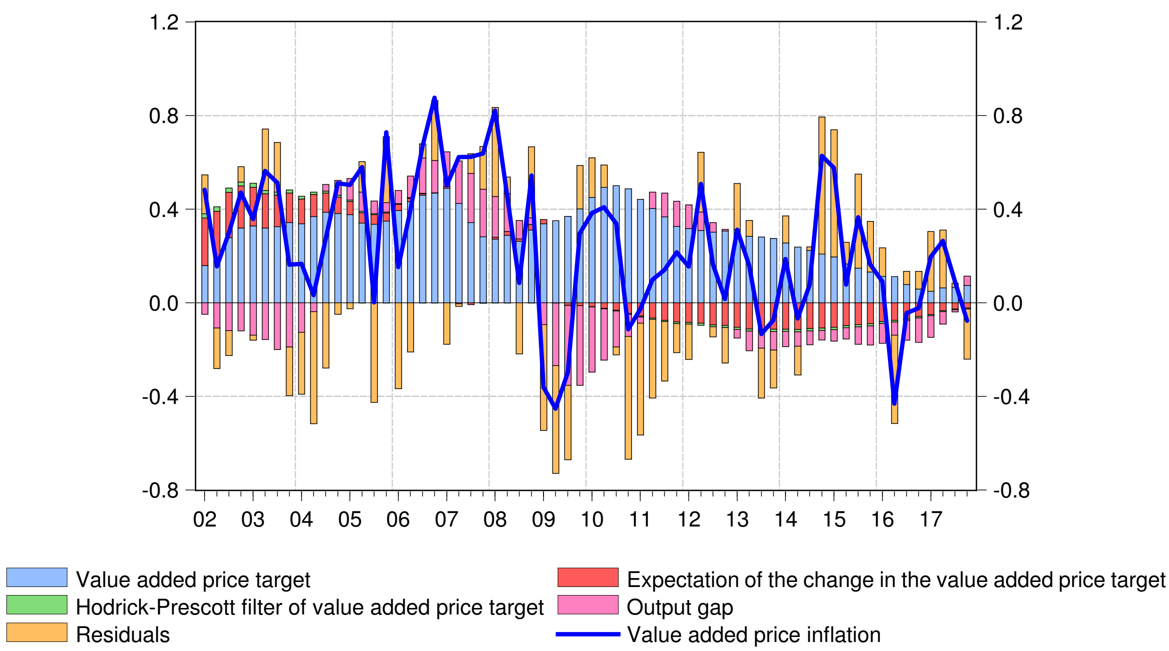
[37] As explained above, $\bar{\pi}_t$ is higher than the mean of the value added price over our estimation sample.

Table 4.4.4: Policy function of the growth rate of the value added price target

Regressors	Policy function $PV(\pi^*_Q)_{t\|t-1}$	Aux. equation $\pi^*_{Q,t}$ (eq 45)	Aux. equation $\tilde{\pi}_{W,t}$ (eq 46)	Aux. equation $\hat{u}_t$ (eq 47)
E-SAT Model variables				
$\hat{y}_{t-1}$	$-1.5 \cdot 10^{-3}$	-	-	0.23
$i_{t-1} - \bar{i}_{t-1}$	$-3.4 \cdot 10^{-3}$	-	-	-
$\pi_{Q,t-1} - \bar{\pi}_{Q,t-1}$	$8.7 \cdot 10^{-4}$	-	-	-
$\hat{y}_{EA,t-1}$	$4.8 \cdot 10^{-4}$	-	-	-
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.00	-	-	-
$\hat{u}_{t-1}$	$-1.1 \cdot 10^{-2}$	-	-	0.946 [0.04]
$\tilde{\pi}_{W,t-1}$	$1.2 \cdot 10^{-2}$	-	0.25 [0.1]	-
$\bar{\pi}^*_{Q,t-1}$	0.44	-	-0.25 [0.1]	-
$\hat{y}_t$	-	-	-	−0.246 [0.07]
$\hat{u}_t$	-	-	−0.04 [0.05]	-
$\tilde{\pi}_{W,t}$	-	0.59 [0.09]	-	-
$\bar{\pi}^*_{Q,t}$	-	0.41 [0.09]	-	-
		$R^2 = 0.48$	$R^2 = 0.02$	$R^2 = 0.92$

Note: standard errors in brackets. $\tilde{\pi}_{W,t} = \pi_{W,t} - \Delta\bar{e}_t$.

Figure 4.4.1: Dynamic contributions, Value added price quarterly inflation, in pp



4.5 Labor market

The labor market block includes (i) labor supply: the wage Phillips curve which defines the link between wage inflation and the expected unemployment gap; and (ii) labor demand: the employment equation, which relates employment to labor costs and aggregate demand in the presence of adjustment costs.

4.5.1 Labor supply

Table 4.5.1: Variables used in section 4.5.1

Notation	**Description**
w_t	Log of gross wage per head of market branches
w_t^m	Log of gross minimum wage
$\pi_{W,t}$	Wage inflation
$\pi_{C,t}$	Growth rate of the consumption deflator
$p_{C,t}$	Log of the consumption price
$\bar{e}_t$	Log of the long-run efficiency
$\Delta\bar{e}$	The long run anchor of the efficiency growth rate
$PV(\hat{u})_{t\|t-1}$	Present value of the expected average of future unemployment gaps
u_t	Unemployment rate
$u_{N,t}$	Long-run trend of the unemployment rate
$\bar{\pi}_t$	Long-run anchor of inflation
E-SAT	See Table 4.2.1
$\hat{u}_t$	Unemployment gap

In the long run the labor supply curve is vertical: the wage elasticity of supply is zero. The unemployment rate is anchored to an exogenously defined long run level, $u_{N,t}$. As explained in subsection 4.3.3, we measure this variable using an estimate provided by the Trade and Sstructural Policies Analysis Division of the Banque de France.

In the short run, the labor supply in FR-BDF is defined by a wage Phillips curve.[38] The equation is augmented with hybrid indexation, which is an important element to recover a significant role for the expected unemployment gap. The wage equation is microfounded. It is derived from the first order condition of agents' optimization problem with respect to leisure. It is important to bear in mind that as FR-BDF is a semi-structural model, the wage equation is not derived jointly with the consumption/saving decision, i.e. we do not impose cross-restrictions between coefficients. We follow Gali et al. (2011), and consider the following New Keynesian wage Phillips curve with indexation:

$$\pi_{W,t} - x_{t-1} = \alpha + \beta E_{t-1}\left[\pi_{W,t+1} - x_t\right] - \lambda(u_t - u_{N,t}) \tag{49}$$

[38] We also considered a wage setting approach but obtaining a significant coefficient of adjustment toward the long run proved difficult.

where the variable x_{t-1} captures the indexation of households unable to optimize their wage in the current period. This indexation variable is determined by the following equation:

$$x_{t-1} = c_1\pi_{C,t-1} + c_2[\pi_{W,t-1} - c_1\pi_{C,t-2}] \tag{50}$$

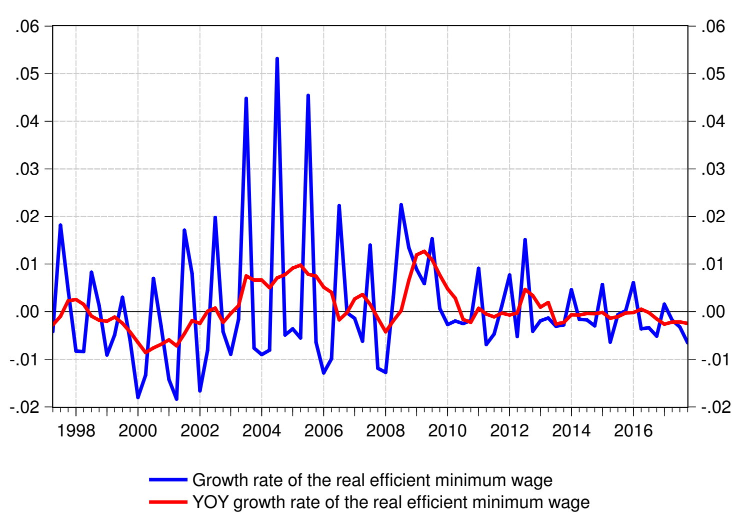
For notations see Table 4.5.1.

Along with indexation, we also add a real efficient minimum wage, computed as the minimum wage per capita adjusted for labor efficiency and long run anchor of inflation. It enters the equation as a year-on-year growth rate (Δ_4) in order to solve the seasonality issue, see Figure 4.5.1. The minimum wage centered by its trend ($\pi_{C,t} + \Delta\bar{e}_t$) is endogenized as follows:

$$w_t^m = \delta_{q1}\left(\pi_{4,C,t} + \Delta_4\bar{e}_t + 0.5\left[\pi_{4,W,t} - \pi_{4,C,t} - \Delta_4\bar{e}_t\right]\right) + \epsilon_t \tag{51}$$

where δ_{q1} is a dummy that takes on a value of 1 in the first quarter of each year; $\pi_{4,C,t}$ is year on year consumer price inflation and $\pi_{4,W,t}$ is year on year wage inflation. Similarly to the government's indexation formula, the specification of this equation takes into account the sensitivity of the minimum wage to consumer price inflation and to wages. However, we made some changes compared to the government formula: we used variables for prices and wages directly available from the model as proxies for variables actually used by the government (consumer price index excluding tobacco of the first income quintile and manual workers basic hourly wage); we included long run anchors in the formula for ensuring the long run stability of the ratio of the minimum wage with respect to the average wage, while this long run stability is ensured by the government through additional positive exogenous shocks referred to as "coups de pouce".

Figure 4.5.1: Growth rate of the real efficient minimum wage



Solving forward equation (49), ignoring the explosive solution and accounting for the balanced growth path, we obtain the wage Phillips curve that we take to the data:

$$\pi_{W,t} = \beta_0 + [\Delta\bar{e}_t + \bar{\pi}_t] + \beta_1(\pi_{C,t-1} - \bar{\pi}_{t-1}) + \beta_2[\pi_{W,t-1} - \Delta\bar{e}_{t-1} - \bar{\pi}_{t-1} - \beta_1(\pi_{C,t-2} - \bar{\pi}_{t-2})] + \beta_3\left(\Delta_4(w^m_{t-1} - e_{t-1}) - \bar{\pi}_{t-1}\right) + \beta_4 PV(\hat{u})_{t-1|t-2} + \epsilon^w_t \tag{52}$$

According to equation (52) presented above, wage inflation is not really on the balanced growth path (BGP) because $\beta_0 \neq 0$. We had to include it in the specification order to center the residuals which were negative on average. We used only one long run trend ($\bar{\pi}_t$) for all inflation types, including wage inflation. This trend was computed using a professional forecast of consumer price inflation, see subsection 3.1.2. The mean of the latter was higher than the mean of any other inflation over the estimation sample period (1997Q2 - 2017Q4), implying that the wage inflation trend $[\Delta\bar{e}_t + \bar{\pi}_t]$ was on average higher than the mean of this series during this period, which leads to negative residuals. In FR-BDF we modify equation (52) so that the constant disappears with time and π^w_t converges toward the balanced growth path.

Note that the present value of the expected sum of future unemployment gaps $PV(\hat{u})_{t-1|t-2}$ enters the equation in $t-1$ and hence is based on the information of $t-2$. In order to compute this variable, we added an auxiliary equation to the core of E-SAT that is in line with Okun's law:

$$\begin{cases} \hat{u}_t = \beta_0 \hat{y}_{t-1} + u_t \\ u_t = \rho u_{t-1} + \epsilon_t \end{cases} \Rightarrow \hat{u}_t = \beta_0(\hat{y}_{t-1} - \rho\hat{y}_{t-2}) + \rho\hat{u}_{t-1} + \epsilon_t \tag{53}$$

The estimates of the policy function, together with those of the auxiliary equation, are available in Table 4.5.2. It is interesting to mention that parameter β_0 from equation (53) is very close to the Okun's law coefficient of the model which was evaluated to be 1/3 and was computed as an elasticity of the unemployment gap with respect to the output gap after a shock to government spending.

Table 4.5.2: Policy function of the expected discounted sum of the future unemployment gaps

VAR Model variables	Policy function $PV(\hat{u})_{t\|t-1}$	Auxiliary equation $\hat{u}_t$
$\hat{y}_{t-1}$	-0.02	-0.27 [0.06]
$i_{t-1} - \bar{i}_{t-1}$	0.11	-
$\pi_{Q,t-1} - \bar{\pi}_{Q,t-1}$	-0.01	-
$\hat{y}_{EA,t-1}$	-0.01	-
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.01	-
$\hat{y}_{t-2}$	-	$-0.27\cdot$ (-0.94)
$\hat{u}_{t-1}$	0.25	0.94 [0.036]

The estimates of equation (52) are presented in Table 4.5.3. The wage Phillips curve strongly influences the relation between the unemployment rate and inflation of the model.

Using estimated parameters, we compute the Phillips slope $\left(\frac{\partial \pi_t}{\partial u_t}\right)$ in partial equilibrium making the following assumptions:

- to account for price indexation, we assume that a 1pp change in $\pi_{W,t}$ results in a 1 pp change in $\pi_{C,t}$.
- we set the Okun's law parameter to 3 (which is consistent with an estimate of the auxiliary equation), i.e. a 1pp increase in the output gap leads to a 1/3pp decrease in the unemployment gap.

$$\frac{\partial \pi_t}{\partial u_t} = \frac{-0.32 \cdot [3 \cdot (-0.02) - 0.25]}{[(1 - 0.24) * (1 - 0.32) - 0.22]} = 0.34 \tag{54}$$

This leads to an elasticity of inflation with respect to the output gap of 0.45 in annual terms, which is close to the estimate of 0.3 obtained by Chatelais et al. (2015).[39]

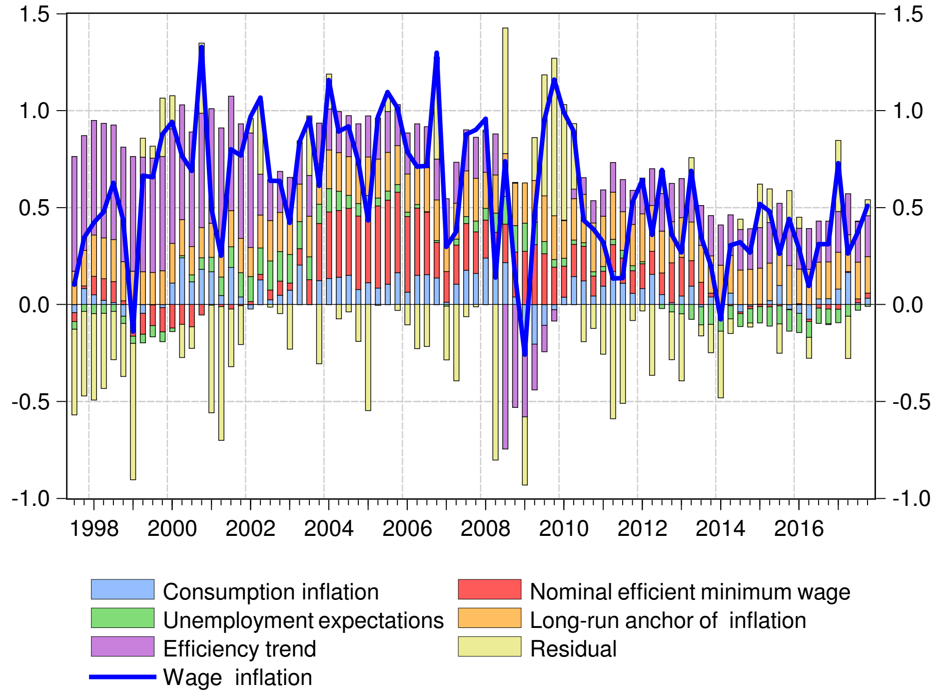
Table 4.5.3: Coefficients and standard errors of the wage Philips curve equation

	Coef.	s.e.
β_0	$-5 \cdot 10^{-4}$	$4 \cdot 10^{-4}$
β_1	0.24	0.1
β_2	0.32	0.1
β_3	0.22	0.1
β_4	-0.32	0.2
$R^2 = 0.18$		

Dynamic contributions Figure 4.5.2 shows how various components contribute dynamically to the variation in wage inflation. Price and wage inflation trends play a major role in describing the dynamics of $\pi_{W,t}$. Expectations also have a significant impact on the wage inflation. They were one of the main forces that drove wage inflation downward after the sovereign debt crisis of 2011. Conversely, the efficient minimum wage pushed wages upward during the period 2004-2014. This upward pressure came partly from the upward harmonization of the various minimum hourly wages, which were brought about by the reduction in working time.

[39] To estimate $\frac{\partial \pi_t}{\partial \hat{y}_t}$ they use quarterly data from the euro area over the period 1997Q1-2014Q4. Note that here the output gap is the difference between observed GDP and potential GDP.

Figure 4.5.2: Dynamic contributions, wage inflation



4.5.2 Labor demand

Table 4.5.4: Variables used in section 4.5.2

Notation	Description
n_t	Total employment of market branches, thousands of persons, in log
$n^*_{S,t}$	Target salaried employment of market branches, thousands of persons, in log
$n_{S,t}$	Salaried employment of market branches, thousands of persons, in log
$n_{NS,t}$	Non-salaried employment of market branches, thousands of persons, in log
q_t	Value added of market branches, volume, in log
$\hat{q}_t$	Market branches output gap
$p_{Q,t}$	Value added price of market branches, in log
$\tilde{w}_t$	Total labor cost per capita, gross wages plus employers' social contributions, value, in log
h_t	Hours per workers, in log
$\bar{e}_t$	Trend labor efficiency, in log
E-SAT	See Table 4.2.1
$\bar{n}^*_t$	HP-trend target salaried employment, in log
$\hat{n}^*_t$	Gap of target salaried employment relative to $\bar{n}^*_t$, in log

We estimate the equation for the employment of market branches based on employees $n_{S,t}$ rather than *total* employment n_t. While both time series are strongly correlated, a specification based on employees performs better than one on total employment.[40]

Table 4.5.5: Coefficient and standard errors for the salaried employment target equation

Coefficient	Estimate	s.e.
b_0	8.85e-2	0.07e-2
$R^2 = 0.98$		

[40] Specifically, when estimating a simple ECM specification augmented by a balanced growth neutrality term (growth rate of filtered employment), an equation based on total employment is better described by a reduced form autoregressive process, while the cointegrating relationship plays a insignificant role. Conversely, an equation based on salaried employment displays a significant cointegrating relationship, which motivates our choice.

Target As detailed in the presentation of the supply-block of FR-BDF, the labor demand equation is derived from the firms' first-order condition for labor which equals marginal productivity to marginal cost of labor, augmented by a mark-up. Hence, the target level of salaried employment $n^*_{S,t}$ is defined by

$$n^*_{S,t} = b_0 + q_t - \bar{e}_t - h_t - \sigma(\tilde{w}_t - p_{Q,t} - \bar{e}_t - h_t) \tag{55}$$

The target level of salaried employment depends on value added, trend efficiency and the hourly efficient real wage (deflated by value added price $p_{Q,t}$) with long run elasticity $\sigma = 0.53$ estimated first step with the business investment equation (see section 4.3 for details).[41] The only estimated parameter is the intercept b_0, which is shown in Table 4.5.5.

Short run equation The short run dynamics of salaried employment are described by a fourth-order PAC equation augmented by the change of the market-branches value added gap $\Delta\hat{q}_t$, to account for an additional Okun's effect in the short run, plus a growth neutrality term, to ensure that salaried employment grows at the same rate as the trend of the employment target in the long run.

The expectation of the present-value change of employment target is decomposed into two components. First, we define the HP-trend of the salaried employment target $\bar{n}^*_t$. Second, we split the expectation into two components: the trend growth rate in the salaried employment $PV(\Delta\bar{n}^*_S)_{t|t-1}$ and the change in the salaried employment gap $PV(\Delta\hat{n}^*_S)_{t|t-1}$. The decomposition of expectations is intended to capture labor hoarding effects in the data. For instance, in the event of a negative transitory shock to the employment target $n^*_{S,t}$, employers would expect the target to return to its long run trend $\bar{n}^*_{S,t}$ and would not reduce their demand for labor as much as in a model without expectations. Finally, ω represents the share of the non-stationary components of the expected changes in the target and is required to ensure growth-neutrality in the long run (see section 3.2 for details). The salaried employment equation (56) was estimated from 1997Q1 to 2017Q4. Results are shown in Table 4.5.6.

$$\begin{aligned}
\Delta n_{S,t} \quad = \quad & \beta_0\big(n^*_{S,t-1} - n_{S,t-1}\big) \\
& + PV(\Delta\bar{n}^*_S)_{t|t-1} + PV(\Delta\hat{n}^*_S)_{t|t-1} \\
& + \beta_1\Delta n_{S,t-1} + \beta_2\Delta n_{S,t-2} + \beta_3\Delta n_{S,t-3} \\
& + (1 - \beta_1 - \beta_2 - \beta_3 - \omega)\Delta\bar{n}^*_{S,t} \\
& + \beta_4\Delta\hat{q}_t + \varepsilon_t
\end{aligned} \tag{56}$$

[41] We use total labor cost $\tilde{w}_t$ from national accounts in the estimation of the target employment equation. The French government introduced the CICE tax credit to decrease cost of labor on medium and low wages, which does appear in national accounts' production subsidies and not in labor cost. In unconditional simulations and forecasting, we instead use the *CICE-adjusted* labor cost. Since total labor cost enters both the employment and VA price equations, two of the three key equations of FR-BDF's supply block, we confirmed that the CICE-adjustment of total labor cost has no significant impact on our estimates; these results are available upon request.

Table 4.5.6: Coefficients and standard errors for the salaried employment short run equation

Coefficient	Estimate	s.e.
β_0	0.06	0.02
β_1	0.87	0.11
β_2	-0.30	0.15
β_3	0.17	0.10
β_4	0.15	0.03
ω	0.26	-

$R^2 = 0.92$

Expectations We construct the expected present value change in the salaried employment gap $PV(\Delta\hat{n}^*_S)_{t|t-1}$ by adding an AR(1) auxiliary equation for $\hat{n}^*_t$ with links to E-SAT core variables for the French economy. The auxiliary equation is:

$$\hat{n}^*_{S,t} = \beta_0\hat{y}_{t-1} + \beta_1(i_{t-1} - \bar{i}_{t-1}) + \beta_2(\pi_{Q,t-1} - \bar{\pi}_{Q,t-1}) + \beta_3\hat{n}^*_{S,t-1} + \varepsilon_t \tag{57}$$

The estimated coefficients of equation (57) and the policy function for $PV(\Delta\hat{n}^*_S)_{t|t-1}$ are shown in Table 4.5.7. The salaried employment gap depends significantly on its lag with coefficient 0.67 and the output gap with coefficient 0.30, which is consistent with the order of magnitude of Okun's Law (see the estimates of the wage Phillips curve auxiliary equation in Table 4.5.2 for example). Conversely, it does not significantly depend on the short-term interest rate and the French VA price inflation gaps but we chose to keep them in the auxiliary equation. Finally, the policy function associated with $PV(\Delta\hat{n}^*_S)_{t|t-1}$ depends on all core variables of E-SAT plus the employment gap $\hat{n}^*_{S,t-1}$. As mentioned earlier, our decomposition of expectations is intended to capture "labor hoarding" effects. In practice, it materializes as a *negative* coefficient associated with $\hat{n}^*_{S,t-1}$. In the case of a negative

Table 4.5.7: Coefficients of policy function and auxiliary equation for expectation of the change in the target employment gap

VAR model	Policy function $PV(\Delta\hat{n}^*_S)_{t\|t-1}$	Auxiliary equation $\hat{n}^*_{S,t}$
$\hat{y}_{t-1}$	0.02	0.30 [0.09]
$i_{t-1} - \bar{i}_{t-1}$	-0.03	0.07 [0.3]
$\pi_{Q,t-1} - \bar{\pi}_{Q,t-1}$	0.02	0.16 [0.13]
$\hat{y}_{EA,t-1}$	0.01	
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.00	
$\hat{n}^*_{S,t-1}$	-0.05	0.67 [0.09]

Note: standard errors are in brackets. $R^2 = 0.82$ for the auxiliary equation.

transitory shock on $\hat{n}^*_{S,t-1}$, it would result in a *positive* effect on $PV(\Delta\hat{n}^*_S)_{t|t-1}$ which would dampen the negative effect on employment.

Finally, the expected present value trend growth rate of the salaried employment target $PV(\Delta\bar{n}^*_S)_{t|t-1}$ is constructed from a calibrated unit-root process for $\Delta\bar{n}^*_{S,t}$ and does not depend on any E-SAT core variables. This choice is motivated by the I(2) nature of HP filtered trends. As a result, the policy function for this expectation is reduced to:

$$PV(\Delta\bar{n}^*_S)_{t|t-1} = \omega\Delta\bar{n}^*_{S,t-1} \tag{58}$$

Dynamic contributions We compute dynamic contributions for equation (56) and show the results in Figure 4.5.3. We observe that movements in target employment account for a large share of the variance of employment, given some delay due to strong persistence in the short run equation. In turn, the expected change in the target employment gap and expected trend growth of target employment significantly dampen the target's contribution, which we interpret as a "labor hoarding" effect. The equation also displays a significant Okun's law effect (through the output gap in E-SAT and in the short run equation) which helps to explain employment dynamics, particularly during the crisis and the recovery. Positive contributions of residuals in 2016 and 2017 mostly reflect the effect of CICE on employment, since we did not adjust total labor cost $\tilde{w}$ for the CICE tax credit at the estimation stage and consequently in these dynamic contributions.[42]

4.6 Demand block

The demand block is composed of four main components: household consumption and investment, business investment and external trade.

4.6.1 Household consumption

In FR-BDF the primary long run driver of household consumption C_t is permanent income, which together with a real interest rate gap determines the consumption target. Permanent income is determined via an expectation term on the ratio of disposable income to output multiplied by the long run output. The interest rate gap is between the households' real bank lending rate and the difference between the long run anchor of the real short rate.

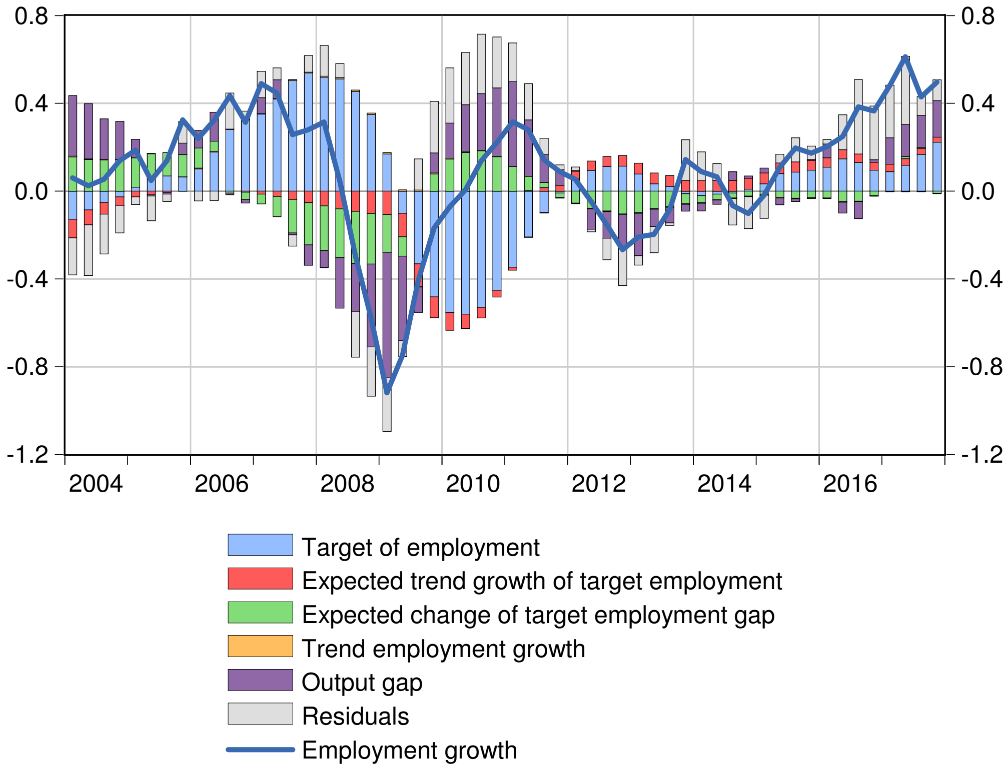
The short run dynamics of consumption are determined with a first order PAC equation augmented with terms representing the behavior of rule-of-thumb consumers (output gap), a wealth effect (change in the interest rate gap) and an indicator that is equal to 1 in the first quarter of 2011, -1 in the second and zero otherwise.[43] The present value of the target is split into expectations regarding the components of the target, which includes an expectation regarding permanent income, i.e. an expectation of an expectation.[44]

[42] As a result, dynamic contributions calculated with a CICE-adjusted total labor cost would display smaller residuals for the same period.

[43] The dummy is the result of a particular government policy in effect at the time, referred to colloquially as "prime à la casse" in French. The purpose of the policy was to subsidize the purchase of new vehicles, like the "cash for clunkers" policy implemented in the United States in 2009.

[44] Note that, as we do not have rational expectations, the expectation of an expectation does not equal the expectation.

Figure 4.5.3: Dynamic contributions (in pp of growth rate), salaried employment



Target The target for households' consumption c_t^* is based on a permanent income term – as described in (60) – represented by a transformation of a standard expectation on the ratio of real disposable household income $y_{H,t}$ to real long run GDP $\bar{y}_t$.[45] Following Campbell & Mankiw (1989), we derive a log-linear consumption equation from the Euler equation and budget constraint of a representative household.[46]

$$c_t^* = \alpha_0 + \mathrm{PV}\left(y_H\right)_{t|t-1} + \alpha_1 \left(r_{LH,t} - (\bar{i}_t - \bar{\pi}_t)\right) \tag{59}$$

The equation for the target, (59), also has an additional term that relates consumption to a real interest rate gap which is an attempt to capture long run effects of interest rates on consumption. The two coefficients of the equation – the constant term α_0 and the sensitivity to the interest rate α_1 – are estimated as -0.16 and -0.95, respectively. The implied intertemporal elasticity of substitution is approximately 0.1.

$$\mathrm{PV}\left(y_H\right)_{t|t-1} = \mathrm{PV}\left(y_H - \bar{y}\right)_{t|t-1} + \bar{y}_t \tag{60}$$

[45]Note that, contrary to FRB/US, the target of consumption depends here on aggregate permanent income with a single propensity-to-consume. In fact, when we tried to distinguish different types of income (wages, assets and transfers) with differentiated propensities, we obtained nonsensical estimates of these propensities. Another difference with respect to FRB/US is the absence of financial and housing wealth from the equation, this variable being insignificant in our estimation on French data.

[46]A detailed note is available on request.

Table 4.6.1: Variables used in section 4.6.1

Notation	Description
c_t	Household consumption, volume (in log)
$y_{H,t}$	Household disposable income, volume (in log)
$r_{LH,t}$	Real household bank lending rate
$\bar{r}_{LH}$	Steady-state of real household bank lending rate
y_t	Gross domestic product, volume (in log)
$\bar{y}_t$	Long run trend of the volume of gross domestic product (in log)
δ_{prime}	"Prime" dummy of 2011
E-SAT	See Table 4.2.1
$\Delta w_{eff,t}$	Growth rate of real efficient wage
$\hat{u}_t$	Unemployment gap

Note : the steady-state real household bank lending rate is defined by a spread over the short-term real interest rate and is equal to $(\bar{i} - \bar{\pi}) + \bar{s}_{LH}$ where $\bar{s}_{LH}$ is the term premium.

Note that the construction of the permanent income term PV $(y_H)_{t|t-1}$ is slightly different to the construction of the other expectations in FR-BDF. In particular, we assume that, due to risk aversion and income uncertainty, the discount factor applied, i.e. the parameter β in the equation is somewhat smaller than in the other cases at roughly 0.95. See Reifschneider (1996) for more details on the derivation of this discount factor. The core of the argument rests on the fact that when optimal consumption – solved from a standard household problem – is related to expected uncertain income, this income stream has to be discounted using not just the real rate of interest as in the perfect foresight case, but also a risk adjustment factor that depends on household risk aversion and the variance of the income stream. As explained in section 6.3, this choice might play a role in the absence of forward guidance puzzle in FR-BDF.

Short run equation The short run dynamics of household consumption are described by a first order PAC equation augmented with several additional terms intended to capture phenomena related to deviations from the permanent income hypothesis. Equation (62) presents this equation and Table 4.6.2 the associated estimated coefficients. The augmenting terms include a term representing non-optimizing behavior by rule-of-thumb households $(\Delta\hat{y}_t)$ and a term intended to capture interest rate effects via the change of the interest rate gap $(\Delta i_{LH,t} - (\Delta\bar{i}_t - \Delta\bar{\pi}_t))$. The third *ad hoc* term is a dummy (δ_{prime}) equal to 1 in the first quarter of 2011, -1 in the second quarter of 2011 and 0 otherwise which reflects a particular government policy (car scrapping allowance), which increased the demand for cars in early 2011. It is included only to improve the empirical fit.

The standard formulation of expectations with respect to changes in the target for consumption has been replaced here with expectation terms on the changes in the various components of the target itself, including an expectation on permanent income, i.e. an expectation of an expectation, which we denote with PV^2 $(y_H - \bar{y})_{t|t-1}$. The stationarity term of the equation has been modified due to this change: as the expectation terms on gaps are already stationary, the term representing the non-stationary component of expectations is now unnecessary and can be omitted.

$$
\begin{aligned}
\Delta c_t \quad = \quad & \beta_0 \left(c^*_{t-1} - c_{t-1}\right) + \beta_1 \Delta c_{t-1} \\
& + \mathrm{PV}^2 \left(y_H - \bar{y}\right)_{t|t-1} \\
& + \alpha_1 \left(\mathrm{PV}\left(r_{LH}\right)_{t|t-1} - \left(\mathrm{PV}\left(\bar{i}\right)_{t|t-1} - \mathrm{PV}\left(\bar{\pi}\right)_{t|t-1}\right)\right) \\
& + \left(1 - \beta_1\right)\left(\Delta(\bar{y}_t) - \Delta\left(y_{H,t} - \bar{y}_t\right)\right) \\
& + \beta_2 \left(\Delta \hat{y}_t\right) + \beta_3 \left(\Delta r_{LH,t} - \left(\Delta \bar{i}_t - \Delta \bar{\pi}_t\right)\right) \\
& + \beta_4 \delta_{prime}
\end{aligned}
\tag{61}
$$

Table 4.6.2: Coefficients and standard errors of the short run equation for household consumption

Coefficient	**Estimate**	**s.e.**
β_0	0.12	0.05
β_1	-0.08	0.09
β_2	0.26	0.11
β_3	-0.71	0.45
β_4	0.007	0.002
$R^2 = 0.54$		

Expectations There are a total of five expectation terms that appear in the various equations of the household consumption block. The most important of these describes expectations regarding the ratio of real disposable income to real output, and is used to model permanent income. The coefficients of the associated policy function and auxiliary equations are presented in Table 4.6.3.

Note that the policy function for permanent income depends on the growth rate of the real efficient wage $\Delta w_{eff,t-1}$ and the unemployment gap $\hat{u}_{t-1}$. These variables were added in order to be able to account for the direct effects of developments in the labor market on permanent income. Note also that the intercept term in the policy function is assumed to be a constant only in the historical sample; in simulation mode, it is assumed to adjust via a learning rule to a level that is consistent with the steady state ratio of income to long run output.

Table 4.6.4 presents the policy function for expected permanent income. Note that the corresponding auxiliary equation is in fact given by the policy function described in Table 4.6.3. This policy function is also dependent on the growth rate of wages and the unemployment gap.

The final expectation terms are described in Table 4.6.6, which presents the policy function and auxiliary equation for the household bank lending rate $i_{LH,t}$, and Table 4.6.7, which

Table 4.6.3: Coefficients of the policy function and auxiliary equation for the expectation of the income-output ratio and auxiliary equations for the real efficient wage and unemployment gap

VAR model	**Policy function**	**Auxiliary equations**		
	$\mathbf{PV}\,(\boldsymbol{y_H - \bar{y}})_{\boldsymbol{t\vert t-1}}$	$\boldsymbol{y_{H,t} - \bar{y}_t}$	$\boldsymbol{\Delta w_{eff,t}}$	$\boldsymbol{\hat{u}_t}$
Constant	-0.29	$-0.49\,(1-0.92)$ [0.009]		
$\hat{y}_{t-1}$	-0.036			0.95 [0.04]
$i_{t-1} - \bar{i}_{t-1}$	-0.085			
$\pi_{t-1} - \bar{\pi}_{t-1}$	0.013			
$\hat{y}_{EA,t-1}$	0.007			
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	-0.004			
$y_{H,t-1} - \bar{y}_{t-1}$	0.39	0.92 [0.036]		
$\Delta w_{eff,t-1}$	0.3	0.32 [0.18]	0.58 [0.00]	
$\hat{u}_{t-1}$	-0.21		-0.08 [0.06]	0.95 [0.04]
$\hat{y}_t$	-			-0.25 [0.06]

Note: standard errors in brackets. $R^2 = 0.91$ for the auxiliary equation of $y_{H,t} - \bar{y}_t$
$R^2 = 0.32$ for $\Delta w_{eff,t-1}$ and $R^2 = 0.92$ for $\hat{u}_{t-1}$

Table 4.6.4: Coefficients of the policy function for the expectation of permanent income

VAR model	**Coefficient** $\mathbf{PV}^{\mathbf{2}}\,(\boldsymbol{y_H - \bar{y}})_{\boldsymbol{t\vert t-1}}$
Constant	-0.043
$\hat{y}_{t-1}$	-0.052
$i_{t-1} - \bar{i}_{t-1}$	-0.012
$\pi_{t-1} - \bar{\pi}_{t-1}$	0.002
$\hat{y}_{EA,t-1}$	0.001
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	-0.001
$y_{H,t-1} - \bar{y}_{t-1}$	0.034
$\mathrm{PV}\,(y_{H,t-1} - \bar{y}_{t-1})$	-0.12
$\Delta w_{eff,t-1}$	0.029
$\hat{u}_{t-1}$	-0.03

presents the policy functions for PV $(\bar{i})_{t|t-1}$ and PV $(\bar{\pi})_{t|t-1}$. The auxiliary equations for the two latter terms are simply the E-SAT core equations as described in Section 3.1.1, implying that appropriately defined policy functions depend only on the two variables themselves.

Note that the policy function of PV $(i_{LH})_{t|t-1}$ is defined in a non-standard way. More specifically, in order to ensure that the process has a zero mean and that the R^2 of the regression is equal to 1, the two interest rates and inflation are centered and the long run anchors of the short rate and inflation are separated from their contemporaneous observations into new explanatory variables. The long-run anchor of the lagged real bank lending rate $(\bar{r}_{LH})$ is set equal to the steady state of the real short-run rate $(\bar{i}-\bar{\pi})$ augmented by a specific spread defined below $(\bar{s}_{LH})$.

The structure of the auxiliary equation for $i_{LH,t}$ is described by (62) and the estimated coefficients are presented in Table 4.6.5. The steady-state term premium $\bar{s}_{LH} = 1.12\%$ (annualized) is the sum of the steady-state premium of the bank lending rate over the 10-year government rate in equation (68) and the steady-state term premium component of the 10 year government rate in equation (96). It can be loosely described as the total premium over the 3 month Euribor.

$$r_{LH,t} = \beta_0 r_{LH,t-1} + (1-\beta_0)\left(\bar{i}_{t-1} - \bar{\pi}_{t-1} + \bar{s}_{LH}\right) + \beta_1\left(i_{t-1} - \bar{i}_{t-1}\right) + \beta_2\left(\bar{\pi}_{t-1} - \pi_{t-1}\right) \quad (62)$$

Table 4.6.5: Coefficients and standard errors of the auxiliary equation for the household real bank lending rate

Coefficient	**Estimate**	**s.e**
β_0	0.88	0.02
β_1	0.12	0.02]
β_2	-0.06	0.02

$R^2 = 0.98$

Table 4.6.6: Coefficients of the policy function for the expectation of the household real bank lending rate

VAR model	**Coefficient PV $(i_{LH})_{t\|t-1}$**
$\hat{y}_{t-1}$	-0.002
$i_{t-1} - \bar{i}$	0.039
$\bar{i}_{t-1} - \bar{i}$	0.012
$\pi_{t-1} - \bar{\pi}$	-0.001
$\bar{\pi}_{t-1} - \bar{\pi}$	0.033
$\hat{y}_{EA,t-1}$	0.001
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.005
$r_{LH,t-1} - \bar{r}_{LH}$	-0.056

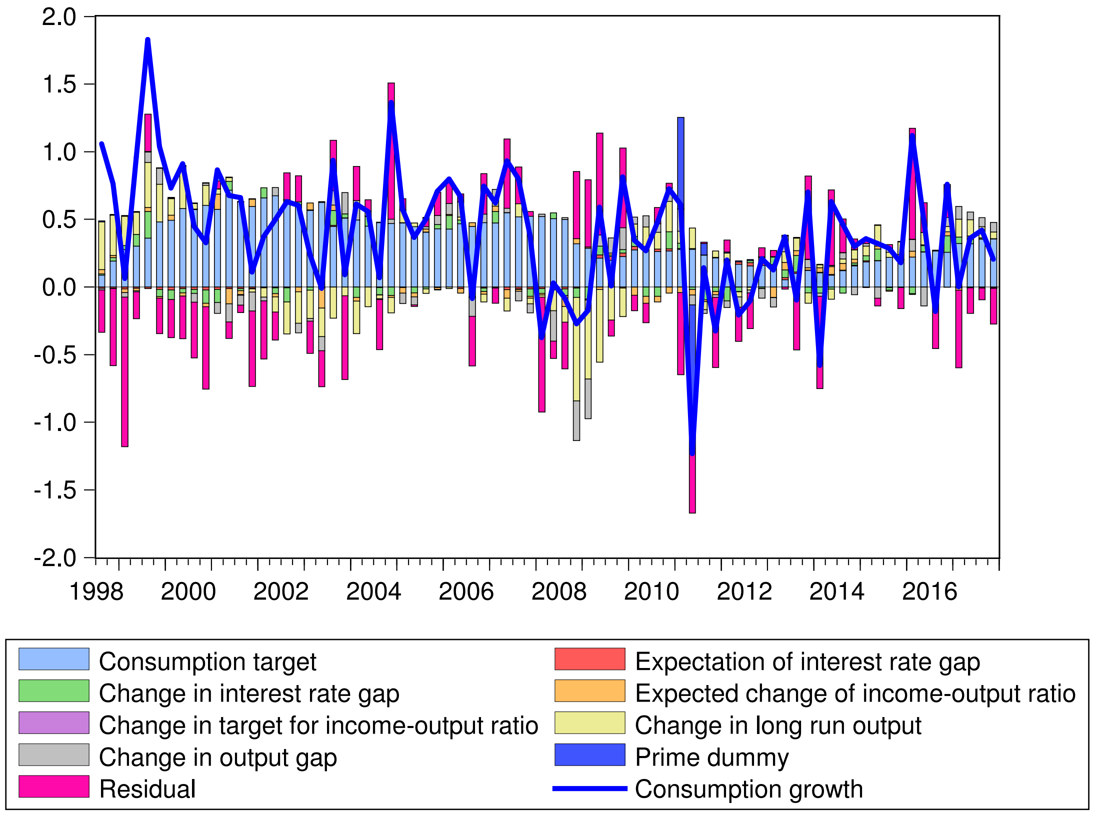
Table 4.6.7: Coefficients of policy functions for expectations of $\bar{i}_t$ and $\bar{\pi}_t$

VAR model	**Policy function** $\mathbf{PV}\left(\boldsymbol{\Delta}\bar{\boldsymbol{i}}\right)_{\boldsymbol{t\|t-1}}$	**Policy function** $\mathbf{PV}\left(\boldsymbol{\Delta}\bar{\boldsymbol{\pi}}\right)_{\boldsymbol{t\|t-1}}$
$\bar{i}_{t-1} - \bar{i}$	-0.011	-
$\bar{\pi}_{t-1} - \bar{\pi}$	-	-0.036

Note: Auxiliary equation defined in E-SAT core equations

Dynamic contributions Figure 4.6.1 describes how the various terms of the short run equation of the consumption block contributed to variation in the growth rate of consumption. They mostly contributed by permanent income. Variation in the long run output also played a role, albeit much smaller; during the financial crisis, however, it had a rather large negative effect. The other terms have individually relatively unimportant effects. Finally, the significant effect of the "prime" dummy, alternating in sign, can be seen quite clearly in the first and second quarters of 2011. Note that the contributions of the terms $i_{LH,t}$, $\bar{i}_t$ and $\bar{\pi}_t$ have been grouped into a single term "interest rate gap", as have the contributions of the corresponding expectations. As shown by these contributions, interest rate changes play in the end a small role in dynamics of French consumption.

Figure 4.6.1: Dynamic contributions, household consumption, in pp of growth rate



4.6.2 Business investment

Table 4.6.8: Variables used in section 4.6.2

Notation	Description
$I_{B,t}$	Investment, volume, NFCs, FCs and sole proprietors
$r_{KB,t}$	Real user cost of capital for firms
$wacc_t$	Weighted average cost of capital
δ_t	Depreciation of firm capital stock
$P_{CI,t}$	Deflator, business investment good
$P_{Q,t}$	Deflator, value added
q_t	Value added, market branches, in log
E-SAT	See Table 4.2.1

What we call here business investment groups investment of different types of firms (non-financial and financial corporations, as well as sole proprietors).[47] In FR-BDF business investment $I_{B,t}$ is strongly based on economic theory. The target is derived from a first order condition for capital for a firm with a CES production function. The short run dynamics of business investment are assumed to follow a second order PAC equation, with an *ad hoc* term representing a business cycle-based demand for firms that do not fully optimize. The standard expectation term in the short run equation is split into the components of the target and defined in terms of gaps from trends in order to obtain a dampening of partial equilibrium impulse responses to shocks that affect expectations.

Target The target of firms' investment $I^*_{B,t}$ is derived from a standard profit maximization problem for a competitive firm that has a CES production function and no investment adjustment costs. The elasticity of the target $I^*_{B,t}$ to the real user cost of capital $r_{KB,t}$ is represented by the parameter σ that is estimated to be 0.53. See section 4.6.2 for details regarding the construction of the user cost. The other coefficient in the equation – α_0 – is estimated to be 0.016. Finally, I^*/K^* is the optimal investment-capital ratio and is assumed to be equal to the historical mean.

$$\log I^*_{B,t} = \alpha_0 + q_t - \sigma \log r_{KB,t-1} + \log \frac{I^*}{K^*} \tag{63}$$

Short run equation The short run dynamics of investment are determined by a second order PAC equation. The estimated coefficients are presented in Table 4.6.9. The expectation term of the equation has been replaced by expectations regarding gap terms of the components of the target – $\hat{q}_t$ and $\hat{r}_{KB,t}$ – where the gap is measured with respect to the long run trends of these variables. As a consequence of this choice, the stationarity term

[47] In particular, investment in social housing is included in this variable, as this type of investment is achieved in France by non-financial companies.

of the equation has been modified. In particular the term representing the non-stationary component of expectations is now unnecessary, as the expectation terms on gaps are already stationary. The last term in the equation is an ad hoc term that represents business cycle-based demand intended to represent the nonoptimizing behavior of some firms.

$$\begin{aligned}\Delta \log I_{B,t} \;=\; & \beta_0 \log \left(\frac{I^*_{B,t-1}}{I_{B,t-1}}\right) + \beta_1 \Delta \log I_{B,t-1} + \beta_2 \Delta \log I_{B,t-2} \\ & + \mathrm{PV}\left(\Delta \hat{q}\right)_{t|t-1} - \sigma \mathrm{PV}\left(\Delta \log \hat{r}_{KB}\right)_{t|t-1} \\ & + \left(1-\beta_1-\beta_2\right)\left(\Delta(\hat{q}_{t-1}) - \sigma \Delta \log(\bar{r}_{KB,t-1})\right) \\ & + \beta_3 \left(\Delta q_{t-1} - \Delta(\bar{q}_{t-1})\right)\end{aligned} \tag{64}$$

Table 4.6.9: Coefficients and standard errors of the short run equation for business investment

Coefficient	**Estimate**	**s.e.**
β_0	0.085	0.029
β_1	0.29	0.14
β_2	0.2	0.1
β_3	0.58	0.36
$R^2 = 0.52$		

User cost of capital for firms The real user cost of capital for firms is determined via equation (65), which relates the user cost to a Weighted Average Cost of Capital (WACC), depreciation δ_t and the ratio of the price of investment goods and the price of value added.[48] The nominal rate is deflated with the term $\mathrm{PV}\left(\pi_Q\right)_{t|t-1}$, i.e. expected value added inflation. Note that there is an implicit assumption of no adjustment costs to investment in the specification of the equation; such costs would imply that the price of investment is not equal to the price of the capital stock, i.e. Tobin's Q is not equal to unity. In addition, in this case a more theoretically rigorous specification would replace value added inflation with inflation of the price of capital. Note that $\mathrm{PV}\left(\pi_Q\right)_{t|t-1}$ is determined with E-SAT; the associated policy function is described in Table 4.6.10. The discount rate used in this computation is $\left(\frac{1}{1+\tilde{i}}\right)^{0.25} \approx 0.998$ where $\tilde{i} \approx 2.29\%$ (annualized) is the sample mean of $\bar{i}_t$ over the full sample.

$$r_{KB,t} = \left[wacc_t + \delta_t - \mathrm{PV}\left(\pi_Q\right)_{t|t-1}\right] \frac{P_{IB,t}}{P_{Q,t}} \tag{65}$$

[48]In this formula, we do not take into account the effect of the capital income tax on the user cost of capital, as is done in Bardaji et al. (2017). This topic is left for further research.

Table 4.6.10: Coefficients of the policy function for expected inflation

VAR model	Coefficient PV $(\pi_Q)_{t\|t-1}$
Constant	0.002
$\hat{Y}_{t-1}$	0.04
$i_{t-1} - \bar{i}_{t-1}$	-0.18
π_{t-1}	0.097
$\bar{\pi}_{t-1}$	0.45
$\hat{y}_{EA,t-1}$	0.019
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	-0.004

Expectations The standard expectation term describing behavior of the investment target that appears in the short run equation has been split into the time-varying components of the target: the value added of the business sector and the user cost of capital for business investment. More specifically, the expectations are formed on gaps of these terms with respect to their trend processes.

The coefficients of the policy functions and the corresponding auxiliary equations for these two terms are presented in Tables 4.6.11 and 4.6.12. The policy function for the gap of the user cost of capital includes the non-standard term $i_{t-2} - \bar{i}_{t-2}$ which has been implemented with the auxiliary variable $x_t = i_{t-1}$. Note that none of the test statistics associated with these estimates have a meaningful interpretation as the dependent variables (the expectations) were created using the independent variables.

Table 4.6.11: Coefficients of the policy function and auxiliary equation for the expectation of market value added

VAR model	Policy function PV $(\Delta\hat{q})_{t\|t-1}$	Auxiliary equation
$\hat{y}_{t-1}$	0.035	
$i_{t-1} - \bar{i}_{t-1}$	-0.101	
$\pi_{t-1} - \bar{\pi}_{t-1}$	0.027	
$\hat{y}_{EA,t-1}$	0.015	
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	-0.002	
$\hat{r}_{KB,t-1}$	0	
$\hat{q}_{t-1}$	-0.071	0.59 [0.048]
$\hat{y}_t$	-	0.61 [0.067]

Note: standard errors in brackets. $R^2 = 0.90$ for the auxiliary equation

Dynamic contributions Figure 4.6.2 describes how the various components contribute dynamically to the variation in business investment. The main driver of variation is the growth of business output, i.e. investment is significantly driven by the business cycle and the

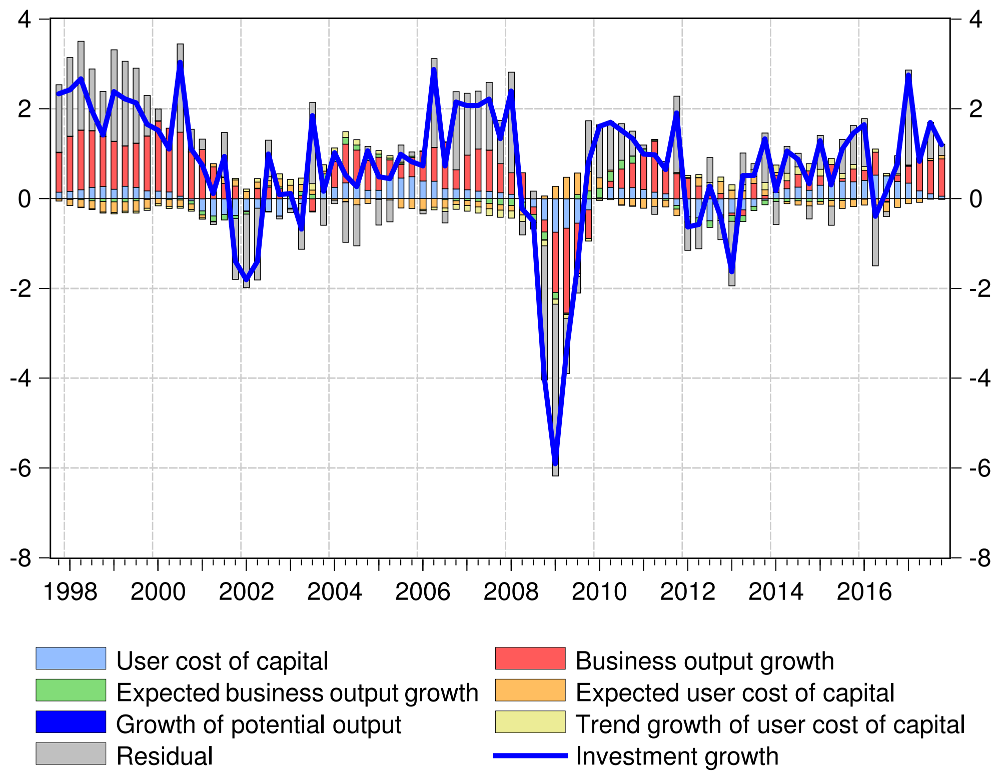
Table 4.6.12: Coefficients of policy function and auxiliary equation for expectation of user cost of capital

VAR model	Policy function PV $(\Delta \log \hat{r}_{KB})_{t\|t-1}$	Auxiliary equation
$\hat{y}_{t-1}$	0	
$i_{t-1} - \bar{i}_{t-1}$	0.24	4.45 [2.36]
$i_{t-2} - \bar{i}_{t-2}$	-0.13	
$\pi_{t-1} - \bar{\pi}_{t-1}$	0	
$\hat{y}_{EA,t-1}$	0.012	
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.038	
$\hat{r}_{KB,t-1}$	-0.055	
$\hat{q}_{t-1}$	0	

Note: standard errors in brackets. $R^2 = 0.63$ for the auxiliary equation

demand that firms are facing. The user cost of capital is also responsible for a notable share of movements in investment, in particular with a negative contribution during the financial crisis. Of the expectation terms, the one related to the user cost is the most important: its impact can be seen especially in 2009, where it has a strong positive contribution. Firms expect the positive shocks to the user cost to be temporary and thus reduce their investment to a lesser extent, i.e. the expectation term has a dampening effect on responses to shocks.

Figure 4.6.2: Dynamic contributions, business investment, in pp of growth rate



4.6.3 Household investment

Table 4.6.13: Variables used in section 4.6.3

Notation	Description
$I_{H,t}$	Household investment, volume
$y_{H,t}$	Household disposable income, volume (in log)
$i_{LH,t}$	Real household bank lending rate
y_t	Gross domestic product, volume (in log)
$\bar{y}_t$	Long run trend of the volume of gross domestic product (in log)
$p_{IH,t}$	Deflator, new housing investment (in log)
$p_{SH,t}$	Deflator, existing housing stock (in log)
$p_{C,t}$	Deflator, household consumption (in log)
$i_{10,t}$	Yield on 10-year French government bonds
E-SAT	See Table 4.2.1

Target The household's target for investment follows (66). The associated estimation results can be found in Table 4.6.14. The desired level of investment by households is assumed to depend on the permanent income term $\mathrm{PV}\left(y_H\right)_{t|t-1}$ and the prices of new $(p_{IH,t})$ and existing $(p_{SH,t})$ housing relative to the price of the consumption good $p_{C,t}$. The final term represents the real user cost of housing investment $\Big(i_{LH,t} - \mathrm{PV}\left(\pi_Q\right)_{t|t-2} + \delta_H\Big)$. δ_H represents the households' depreciation rate and is calibrated at 1.8% per year.

$$
\begin{aligned}
\log I^*_{H,t} \quad = \quad & \log \gamma_0 + \mathrm{PV}\left(y_H\right)_{t|t-1} \\
& + \gamma_1 \left(p_{IH,t-1} - p_{C,t-1}\right) + \gamma_2 \left(p_{SH,t-2} - p_{C,t-2}\right) \\
& + \gamma_3 \log \Big(i_{LH,t-2} - \mathrm{PV}\left(\pi_Q\right)_{t-2|t-3} + \delta_H\Big)
\end{aligned}
\tag{66}
$$

Table 4.6.14: Coefficients and standard errors of the long run equation for household investment

Coefficient	Estimate	s.e.
γ_0	0.005	0.001
γ_1	-2.2	0.11
γ_2	0.55	0.036
γ_3	-0.071	0.023

$R^2 = 0.82$

Short run equation The short run dynamics of household investment are described by (67), while Table 4.6.15 presents the relevant estimation results. We assume that $m = 2$, i.e. implying a single lag of household investment on the right hand side of the equation. Expectations regarding changes in the target have been split into gap and trend components. As the trend component is present in the specification, the standard PAC specification for ensuring growth neutrality is applied, with ω, the share of the nonstationary component in expectations present in the equation.[49] The equation also contains an *ad hoc* term, the contemporaneous change in the output gap, to account for liquidity contrained households and direct effects of demand and the contemporaneous change in the deflator of existing housing, centered by its trend, to account for short-term dynamics of housing prices.

$$\begin{aligned}\Delta \log I_{H,t} \;&=\; \beta_0 \log\left(\frac{I^*_{H,t-1}}{I_{H,t-1}}\right) + \beta_1 \Delta \log I_{H,t-1} \\ &\quad + \mathrm{PV}\left(\Delta \log \hat{I}^*_H\right)_{t|t-1} - \mathrm{PV}\left(\Delta \log \bar{I}^*_H\right)_{t|t-1} \\ &\quad + (1-\beta_1-\omega)\,\Delta \log \bar{I}^*_{H,t} \\ &\quad + \beta_2 \Delta \hat{y}_t + \beta_3 \left(\Delta p_{SH,t} - \Delta \bar{p}_{SH,t}\right)\end{aligned} \tag{67}$$

Table 4.6.15: Coefficients and standard errors of the short run equation for household investment

Coefficient	Estimate	s.e.
β_0	0.056	0.019
β_1	0.62	0.069
β_2	0.34	0.20
β_3	0.32	0.09
ω	0.36	-

$R^2 = 0.87$

Expectations Expectations regarding changes in the target for household investment have been split into two different components: the trend of the target and a gap term measuring deviations from this trend. The estimation results for the policy functions and auxiliary equations relating to the second term can be found in Table 4.6.16. The policy function for the change in the trend component is a simple AR(1) with a coefficient equal to 0.25, the share of the nonstationary component in expectations.

Bank lending rate for households The dynamics of $i_{LH,t}$ are described by (68). Change in the bank lending rate is related to the difference between the rate and the 10-year government rate via an error correction mechanism, with α_0 representing a long run premium

[49] See section 3.2.1 for details.

Table 4.6.16: Coefficients of policy functions and auxiliary equations for expectations of the gap of the household investment target

VAR model	**Policy function** $\mathbf{PV}\left(\boldsymbol{\Delta}\log\hat{\boldsymbol{I}}^{*}_{\boldsymbol{H}}\right)_{\boldsymbol{t|t-1}}$	**Auxiliary equation**
$\hat{y}_{t-1}$	0.029	0.38 [0.26]
$i_{t-1}-\bar{i}_{t-1}$	-0.15	-0.89 [0.96]
$\pi_{t-1}-\bar{\pi}_{t-1}$	0.035	0.49 [0.54]
$\hat{y}_{EA,t-1}$	0.004	-
$\pi_{EA,t-1}-\bar{\pi}_{EA,t-1}$	-0.012	-
$\log\hat{I}^{*}_{H,t-1}$	-0.044	0.71 [0.096]

Note: standard errors in brackets. $R^2 = 0.62$ for the auxiliary equation

over the public rate.

$$\Delta i_{LH,t} = \alpha_1\left(i_{LH,t-1} - i_{10,t-1} - \alpha_0\right) + \alpha_2\Delta i_{10,t} + \alpha_3\Delta i_{10,t-1} + \alpha_4\Delta i_{LH,t-1} + \alpha_5\Delta i_{10,t-2} \quad (68)$$

Table 4.6.17: Coefficients and standard errors of the equation for household bank lending rate

Coefficient	**Estimate**	**s.e.**
α_0	0.001	0.0005
α_1	-0.047	0.022
α_2	0.075	0.031
α_3	0.26	0.038
α_4	0.66	0.077
α_5	-0.19	0.071

$R^2 = 0.52$

Price of housing stock The dynamics of the price of the existing housing stock are given by (69), which is a relatively simple $AR(2)$ process in the change of the logarithm of the price level with a constrained intercept.

$$\Delta p_{SH,t} = \rho_0\Delta p_{SH,t-1} + \rho_1\Delta p_{SH,t-2} + \left(1-\rho_0-\rho_1\right)\bar{pi}_t \quad (69)$$

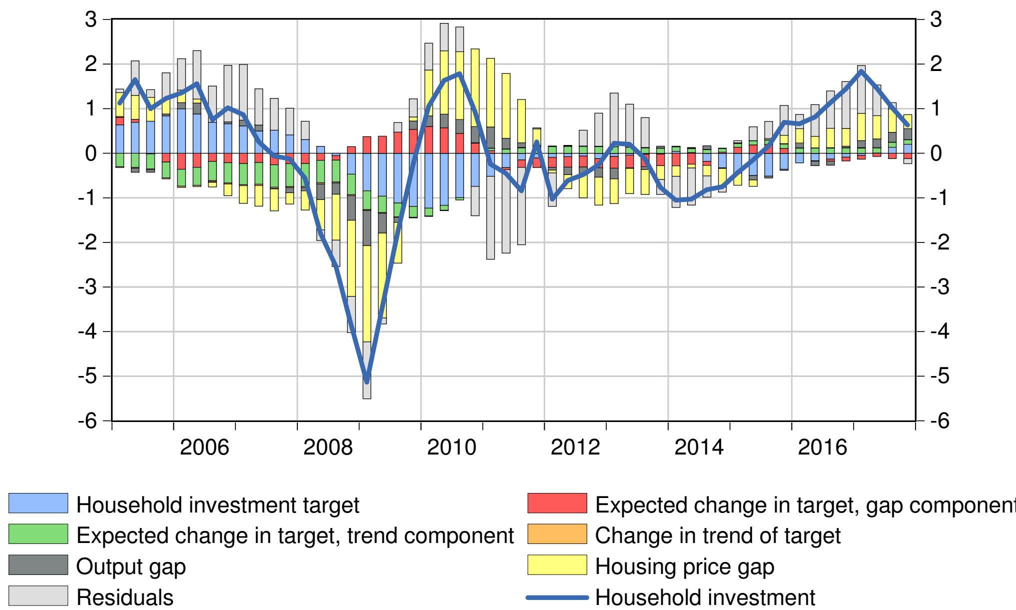
Dynamic contributions Figure 4.6.3 shows how household investment is mostly driven by the variation in the target, which follows the business cycle with a delay. Expectations, in particular the gap component, also have a notable role, but in the opposite direction *vis-à-vis*

Table 4.6.18: Coefficients and standard errors of the equation for the price of housing stock

Coefficient	Estimate	s.e.
ρ_0	0.48	0.1
ρ_1	0.43	0.1
$R^2 = 0.72$		

the business cycle and household income, reflecting a dampening effect. Finally, the housing price gap and, to a lesser extent, the *ad hoc* output gap term have an amplifying effect on household investment, e.g. driving it further down during the Great Recession and pusing it further up during the immediate recovery.

Figure 4.6.3: Dynamic contributions, household investment, in pp of growth rate



4.6.4 External trade

As we show below, we were able to obtain high estimates for the price elasticities in equations of exports and non-energy imports excluding energy, thanks to the inclusion of the weight of emerging countries in the first case and a goods variety indicator in the second. As explained in section 5.1, these elasticities play a key role in the long run convergence of the model.

Exports The real export equation is estimated as a one-step ECM. The estimation period is 2000Q2-2017Q4.

Table 4.6.19: Variables used in section 4.6.4

Notation	Description
X_t	Exports (volume)
WS_t	World supply for France (volume)
$P_{X,t}$	Export price
$P_{MO,t}$	Import price, other than energy
$P_{CX,t}$	Foreign competitors price' (export side)
ω_t	Weight of emerging countries (in log)
$\Delta\bar{q}$	Long run anchor of the output growth rate
$M_{O,t}$	Non-energy imports
$Q_{SM,t}$	Value added, market sector
$M_{NRJ,t}$	Energy imports (volume)
$P_{MNRJ,t}$	Energy import price
$P_{Q,t}$	Value added price
$\bar{T}_t$	Time-varying trend
$D_{MNRJ,t}$	Import intensity-adjusted measure of aggregate demand (IAD) for energy imports
$D_{MO,t}$	Import intensity-adjusted measure of aggregate demand (IAD) for imports other than energy
C_t	Household consumption, volume
$I_{B,t}$	Investment, volume, NFC, FC and sole proprietor
$I_{H,t}$	Household investment, volume
$C_{G,t}$	Government consumption, volume
$I_{G,t}$	Government investment, volume

Target World demand is the only regressor in the equation that has a non-zero growth rate.[50] Hence, we need a unit coefficient in front of the latter in order to satisfy the balanced growth path condition. The ratio $\frac{P_{X,t}}{P_{cx,t}}$ in the target is used as a price competitiveness indicator specific to exports. The weight of emerging countries is also used here in some sense to reveal the competitiveness of French producers not captured by the price difference.[51]

$$x_t^* = \beta_0 + d_{W,t} + \beta_1(p_{X,t} - p_{CX,t}) + \beta_2\omega_t \tag{70}$$

For notations, see Table 4.6.19. Estimated coefficients are shown in Table 4.6.20.

Table 4.6.20: Estimates and calibrated parameters, exports

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	11.93	0.02	0.83	0.05
β_1	-1.27	0.41	-0.15	0.06
β_2	-0.63	0.07	-	-
R^2 = 0.69 (1 step estimation)				

Short run equation

$$\Delta x_t = \beta_0 \Delta d_{W,t-1} + (1-\beta_0)\Delta\bar{q} + \beta_1\left[x_{t-1} - x_{t-1}^*\right] + \epsilon_t \tag{71}$$

Dynamic contributions Variation in the world demand for French goods is the main contributor to French export fluctuations, see Figure 4.6.4. The weight of emerging countries in world trade helps explaining the underperformance of French market shares from 2003 to 2014.

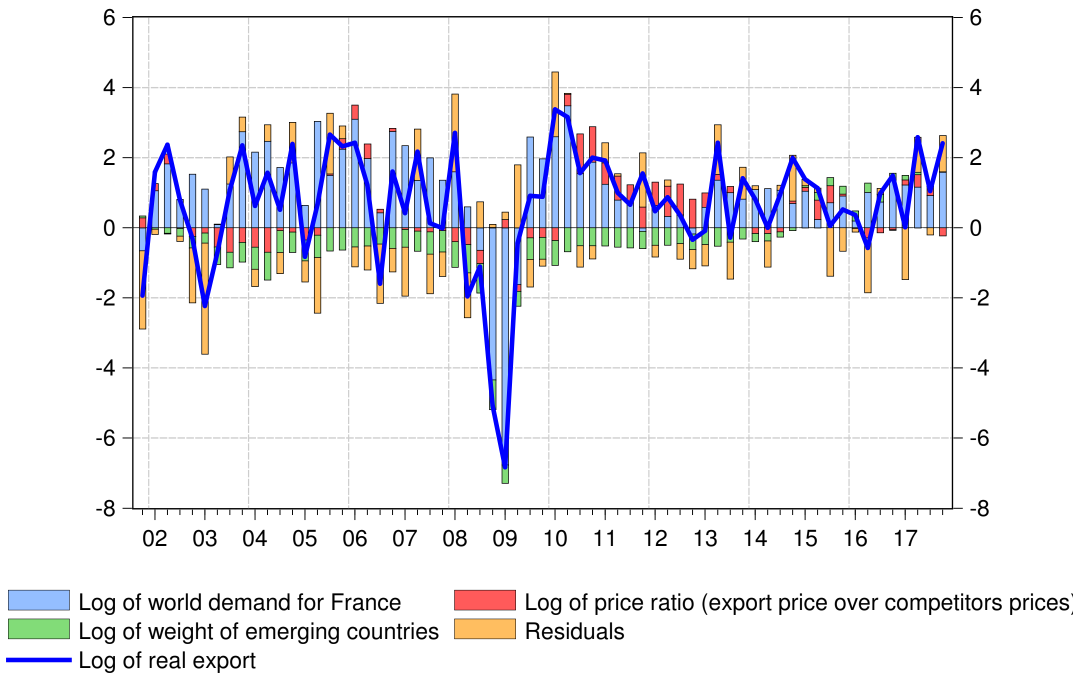
Imports We model separately volumes for energy and other imports. The choice to split the total import volume and price is due to the heterogeneity in coefficients of adjustment (for price) and elasticity of substitutions (for volumes). Total imports are then simply modeled through a chained price aggregation.

Import intensity-adjusted measures of aggregate demand (IAD) for non-energy imports and energy play an important role in the equations of imports. They are computed with weights based on input-output tables including the option to re-export imported goods or services, see equations (72) and (73).

[50] By world demand we mean the weighted sum of imports by trade partners. For more details about computational aspects of these variables see Hubrich & Karlsson (2010).

[51] This variable captures the fact that a share of foreign demand is not "addressed" to France but is met by emerging economies whose competitiveness is not adequately measured by our price competitiveness indicator, possibly because these countries expand at the extensive margin (new varieties, new destinations) rather than at the intensive margin (increased price competitiveness on existing markets and products).

Figure 4.6.4: Dynamic contributions, Export (in pp of growth rate)



$$D_{MO,t} = 0.194C_t + 0.094C_{G,t} + 0.252I_{B,t} + 0.197I_{G,t} + 0.150I_{H,t} + 0.305X_t \quad (72)$$

$$D_{MNRJ,t} = 0.037C_t + 0.012C_{G,t} + 0.013I_{B,t} + 0.014I_{G,t} + 0.016I_{H,t} + 0.026X_t \quad (73)$$

Non-energy imports This equation is estimated as a two step ECM. The estimation sample is 2001Q2-2014Q4.

Target

$$m^*_{O,t} = \beta_0 + d_{MO,t} + \beta_1 (p_{X,t} - p_{MO,t}) + \beta_2 (ws_t - q_{N,t}) \quad (74)$$

The variable $ws_t - q_{N,t}$ is a proxy for the relative variety of goods in the world with respect to France, i.e. the variety of foreign goods relative to the variety of French supply. The former is proxied by the weighted average of exports of countries supplying France (weights are their share in French imports). The construction is symmetrical to that of world demand for French goods (see footnote 50). The latter is proxied by the French long run value added of market branches. It helps in attributing a substantial role to price competitiveness. Price competitiveness is proxied here by the relative price of exports with respect to the price of non-energy imports, as we consider the price of exports as a better proxy of the price of tradable goods produced locally than the value added price.

Short run equation

$$\Delta m_{O,t} = \beta_0 \Delta d_{MO,t-1} + (1-\beta_0)\Delta\bar{y} + \beta_1 \left[m_{O,t-1} - m^*_{O,t-1} \right] + \epsilon_t \tag{75}$$

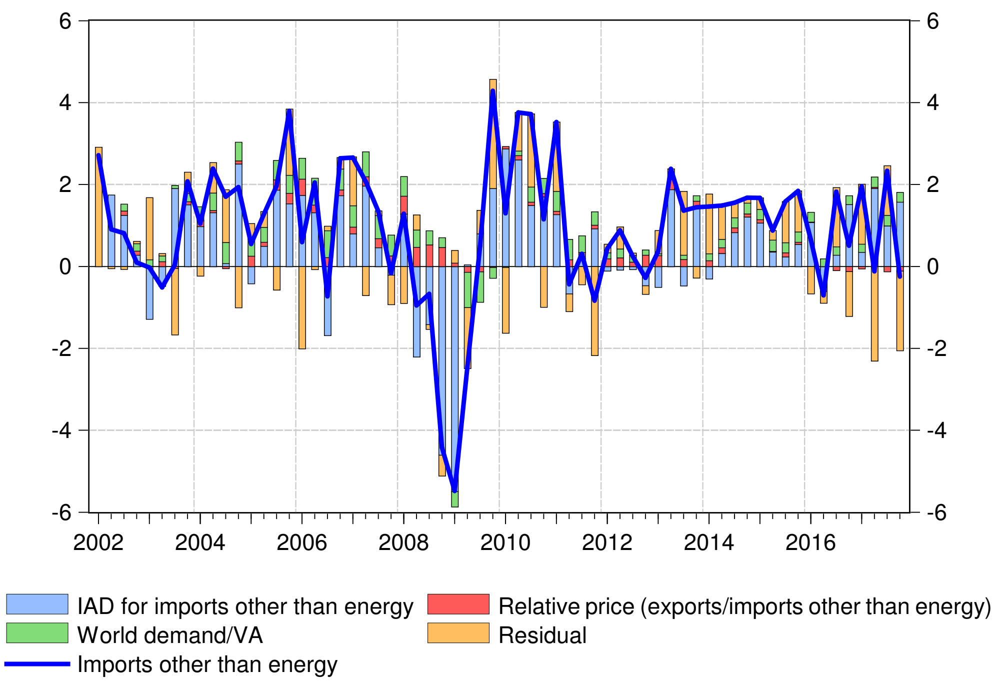
For notations, see Table 4.6.19. Estimated coefficients are shown in Table 4.6.21.

Table 4.6.21: Estimates and calibrated parameters, non-energy imports

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	2.79	0.3	1.91	0.13
β_1	1.11	0.19	-0.19	0.1
β_2	0.39	0.04	-	-
	$R^2 = 0.98$		$R^2 = 0.67$	

Dynamic contributions The main driver of the non-energy import dynamics is the import intensity-adjusted measure of aggregate demand for such imports, see Figure 4.6.5.

Figure 4.6.5: Dynamic contributions, non-energy imports (in pp of growth rate)



Energy imports (volume) This equation is estimated as a one-step ECM, i.e. target and short run equations are estimated simultaneously. The estimation sample is 2000Q1-2017Q4.

Target

$$m^*_{NRJ,t} = d_{MNRJ,t} + \beta_0 \left(p_{MNRJ,t} - p_{Q,t}\right) + \beta_1 + \beta_2 \bar{T}_t \tag{76}$$

Short run equation

$$\Delta m_{NRJ,t} = (1 - \beta_0 - \beta_1)\Delta\bar{y} + \beta_0 \Delta d_{MNRJ,t} + \beta_1 \Delta m_{NRJ,t-1} + \beta_2 \left[m_{NRJ,t-1} - m^*_{NRJ,t-1} \right] + \epsilon_t \tag{77}$$

In order to recover a meaningful coefficient β_0 of the price ratio in the target equation (76), we have to estimate both equations in one step without any long run restrictions, i.e. allowing the constant in the short run equation to be freely estimated, which yields $\beta_0 = -0.16$, see Table 4.6.22. In the next step we calibrate β_0 to -0.16 and estimate other parameters of (77).

Table 4.6.22: Estimates and calibrated parameters, energy imports (in pp of growth rate)

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	-0.16	0.08	1.17	1.15
β_1	0.35	0.12	-0.36	0.11
β_2	-0.004	0.002	-0.12	0.08

$R^2 = 0.26$ (One-step estimation)

Note: β_0 was estimated separately. See text for details.

Dynamic contributions Since the R^2 of equation (77) is very small indicating a poor empirical fit of this equation, most of the dynamics of the growth rate of energy imports come from residuals. Plotting dynamic contributions is in this case unnecessary.

4.7 Demand deflators

We model eight deflators with econometric equations: household consumption and investment deflators, government consumption deflator, corporate investment deflator, export deflator, import excluding energy and energy deflators, total import deflator and value added deflator of market branches.[52] For all, except the last two, we use common transverse principals. First, their long run relation is modeled as a linear combination of market value added and import prices. The market value added deflator is described in section 4.4. The import deflator is computed as an accounting identity: value of imports over volume of imports. The latter is defined using annual prices of imports excluding energy and annual prices of energy. Second, the weight of the import price is defined using weights of the import content of demand (henceforth, IAD weights). These weights are calculated using input-output tables including a re-exporting option of imported goods or services. Third, the deflators are modeled as error correction equations.

In the case of the household consumption deflator, the corporate investment deflator and the household investment deflator, we model their homologies excluding VAT, which are linked to the former through a tax rate which is exogenous. For example, let the household consumption deflator including and excluding VAT be respectively $\pi_{C+vat,t}$ and $\pi_{C,t}$, and the tax rate – τ_C, then their relation is simply presented as:

$$P_{C+vat,t} = (1 + \tau_C)P_{C,t} \tag{78}$$

4.7.1 Household consumption deflator excluding VAT

Table 4.7.1: Variables used in section 4.7.1

Notation	Description
$\pi_{C,t}$	Consumer price inflation excluding VAT
$p^*_{C,t}$	Target of consumer price (in log)
$p_{C,t}$	Consumer price (in log)
$p_{Q,t}$	Value added price of market branches (in log)
$p_{M,t}$	Import price (total) (in log)
$P_{MNRJ,t}$	Energy import price
$\bar{P}_t$	Exogenous trend price
$\bar{\pi}_t$	Long run anchor of inflation
T_t	Time-varying trend

The household consumption deflator plays a significant role in FR-BDF. Households use it to index their gross wage and the government uses it to index the minimum wage, see equation (52). It is also used to deflate disposable income and many items of public spending are indexed on it. Its equation is estimated as an ECM in two steps on the 2000Q1 - 2017Q4 sample.

[52] Sometimes in the text we refer to the value added price (deflator) of market branches simply as the (market) value added price (deflator).

Target In order to center the residuals we had to include a time-varying trend (T_t) which would become a constant at some point so that the deflator converges to the balanced growth path. Parameter β_0 is calibrated to be consistent with the corresponding IAD weight, i.e. with the share of imports in consumption. For notations see Table 4.7.1. Estimated and calibrated parameters are in Table 4.7.2.

$$p^*_{C,t} = (1-\beta_0)p_{Q,t} + \beta_0 p_{M,t} + \beta_1 T_t + \beta_2 \tag{79}$$

Table 4.7.2: Estimates and calibrated parameters, household consumption deflator

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.23	-	0.63	0.07
β_1	$5.4 \cdot 10^{-4}$	$0.3 \cdot 10^{-4}$	0.16	0.07
β_2	-0.043	0.002	0.05	0.003
β_3	-	-	-0.05	0.03
	$R^2 = 0.82$		$R^2 = 0.85$	

Short run equation In the short run, in order to obtain a non-linear effect of oil price shocks, we use the absolute variation in the energy import price – normalized by the exogenous trend price $\bar{P}_t$ to preserve the BGP of FR-BDF – rather than the growth rate of the energy import price.[53]

$$\begin{aligned}\pi_{C,t} = (1-\beta_0-\beta_1)\bar{\pi}_{Q,t} + \beta_0\pi_{Q,t} + \beta_1\pi_{Q,t-1} + \beta_2\Delta(P_{MNRJ,t}/\bar{P}_t) \\ +\beta_3\left[p_{C,t-1} - p^*_{C,t-1}\right] + \epsilon_t\end{aligned} \tag{80}$$

Dynamic contributions See Figure 4.7.1. The value added price played an important role mainly before 2008. After that, the dynamics of consumption inflation were mainly driven by the energy import price from the short run equation.

4.7.2 Business investment deflator

The business investment deflator $\pi_{IB,t}$ influences the model through the real cost of capital. It is estimated as a one-step ECM: the target and the short run equations are estimated simultaneously.

Target The parameter β_0 is calibrated to be consistent with IAD weights, i.e. with the share of imports in business investment.. For notations see Table 4.7.3. Estimated and calibrated parameters are in Table 4.7.4. The estimation period is 1994Q4-2017Q4.

[53]This exogenous trend price is normalized to 1 at the base year 2014 and with a constant growth rate equal to FR-BDF's steady-state inflation rate $\bar{\pi}$.

Figure 4.7.1: Dynamic contributions, consumer price inflation (in pp of growth rate)

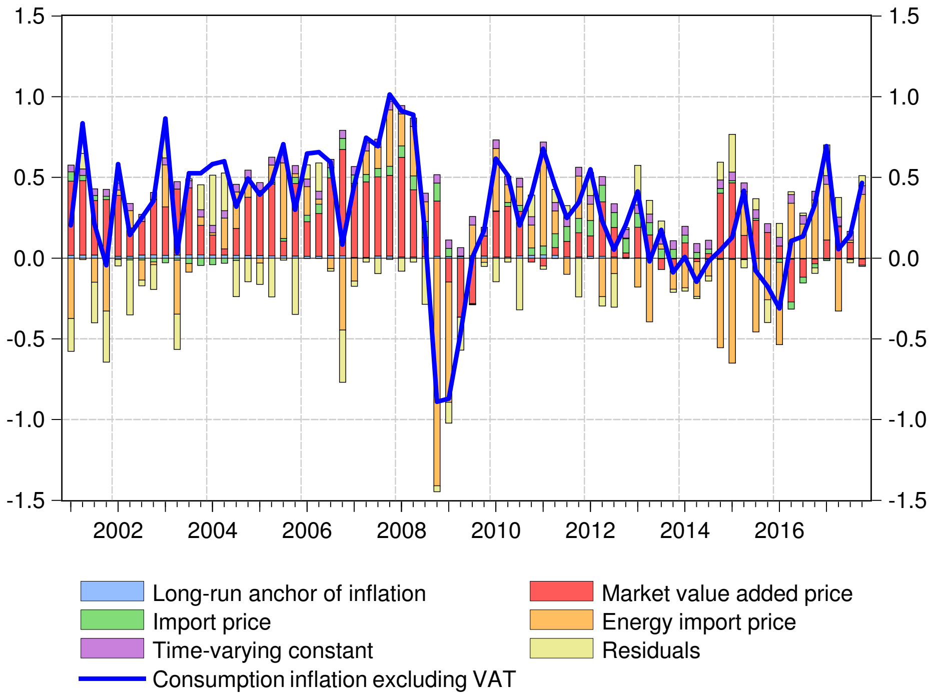


Table 4.7.3: Variables used in section 4.7.2

Notation	Description
$\pi_{IB,t}$	Business investment price inflation
$p^*_{IB,t}$	Target of business investment price (in log)
$p_{IB,t}$	Business investment price (in log)
$p_{Q,t}$	Value added price of market branches (in log)
$p_{MO,t}$	Import price (excluding energy)(in log)
$p_{MNRJ,t}$	Energy import price (in log)
$\bar{\pi}_{Q,t}$	Long run trend of the value added price inflation
$\pi_{M,t}$	Import price inflation (total)

$$p^*_{IB,t} = (1-\beta_0)p^*_{Q,t} + \beta_0\left[(1-\beta_1)p_{MO,t} + \beta_1 p_{MNRJ,t}\right] \tag{81}$$

Short run equation The short run equation is estimated as an ECM. The estimation period is 1995Q1 - 2017Q4.

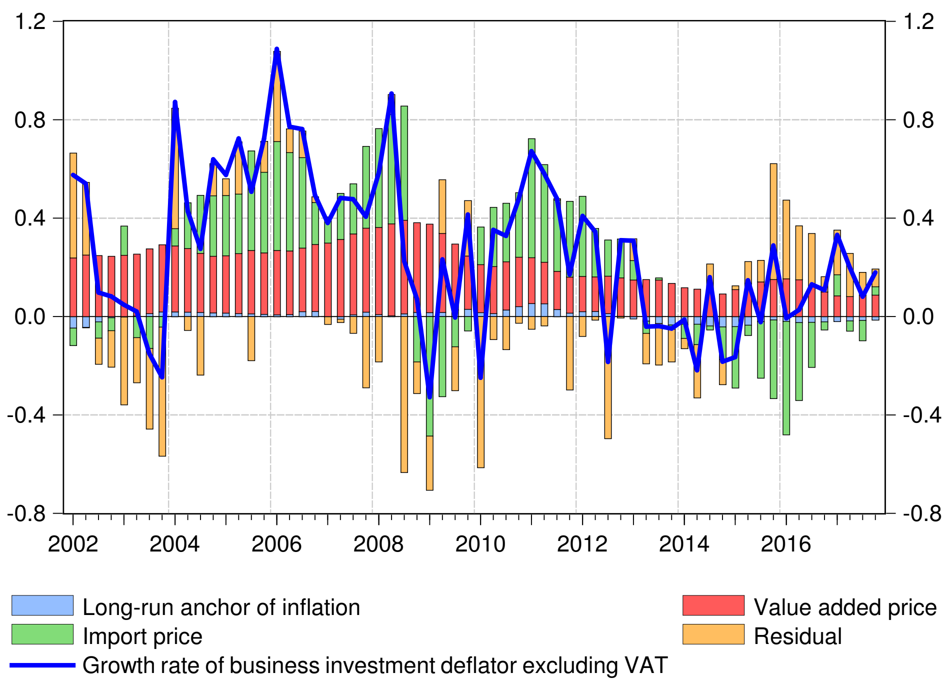
$$\pi_{IB,t} = (1-\beta_0-\beta_1)\bar{\pi}_{Q,t} + \beta_0\pi_{M,t} + \beta_1\pi_{IB,t-1} + \beta_2\left[p_{IB,t-1} - p^*_{IB,t-1}\right] + \epsilon_t \tag{82}$$

Table 4.7.4: Estimates and calibrated parameters, business investment deflator excluding VAT

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.27	-	0.11	0.03
β_1	0.25	0.01	0.42	0.1
β_2	-	-	-0.06	0.03
$R^2 = 0.38$	(1 step estimation)			

Dynamic contributions The import price played an important role in explaining the hikes in the deflator from 2004 to 2008, and from 2010 to 2012, ensuring a good econometric fit, see Figure 4.7.2. Note that the import price here also includes energy and non-energy import prices from the short run equation.

Figure 4.7.2: Dynamic contributions, business investment deflator excluding VAT (in pp of growth rate)



4.7.3 Household investment deflator

This deflator, $\pi_{IH,t}$, is estimated as a two-step ECM. The estimation period is 1995Q1-2017Q4.

Table 4.7.5: Variables used in section 4.7.3

Notation	**Description**
$\pi_{IH,t}$	Household investment deflator
$p^*_{IH,t}$	Target of household investment price (in log)
$p_{IH,t}$	Household investment price (in log)
$p_{Q,t}$	Value added price of market branches (in log)
$p_{M,t}$	Import price (excluding energy) (in log)
$p_{immo,t}$	Price of existing housing stock (in log)
$\delta_{99-08,t}$	Dummy 1: 1 during 1999-2008, 0 otherwise
$\delta_{99Q4,t}$	Dummy 2: 1 during 1999Q4, 0 otherwise
$\delta_{08Q4,t}$	Dummy 3: 1 during 2008Q4, 0 otherwise
$T_{08Q3,t}$	Time-varying trend from 1995Q1, a constant after 2008Q3.
$\bar{\pi}_{Q,t}$	Long run trend of the value added price inflation
$\pi_{immo,t}$	Growth rate of price of existing housing stock

Target The target of this deflator is a weighted average of the value added price and the total import price. The parameter β_1 is calibrated to be consistent with IAD weights. For notations, see Table 4.7.5. In order to center the residuals a time-varying trend $T_{08q3,t}$ is introduced. After 2008Q3 $T_{08q3,t}$ is equal to the last value of the series. Estimated and calibrated parameters are in Table 4.7.6.

$$p^*_{IH,t} = \beta_0 + (1 - \beta_1)p_{Q,t} + \beta_1 p_{M,t} + \beta_2 T_{08q3,t} + \beta_3 \delta_{99-08,t} \tag{83}$$

Table 4.7.6: Estimates and calibrated parameters, household investment deflator

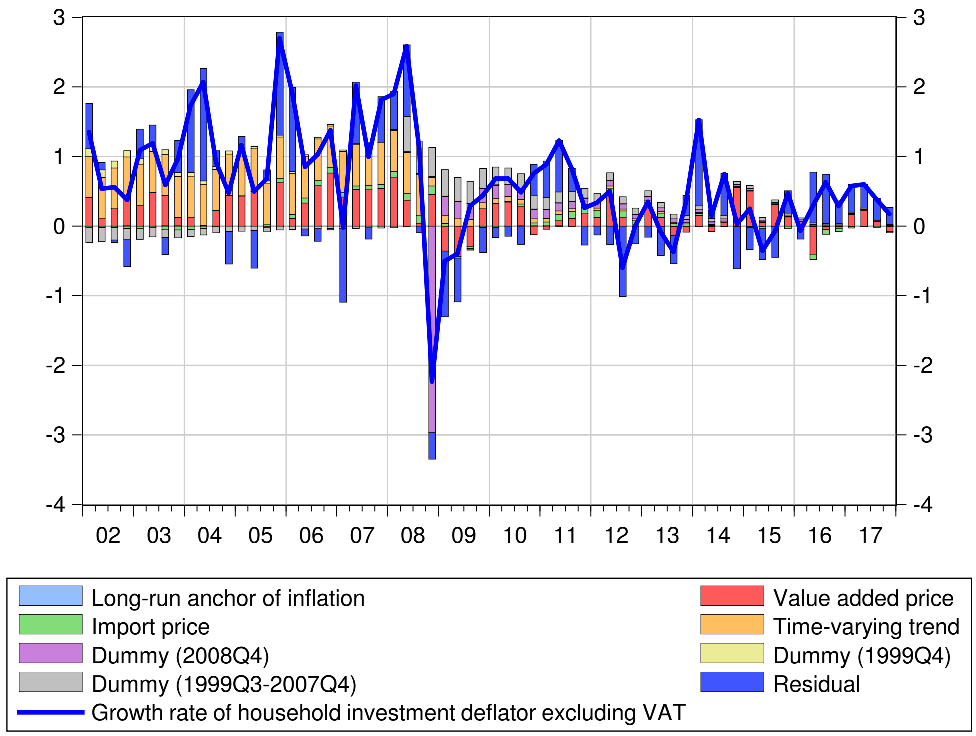
	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	-0.35	0.005	0.9	0.18
β_1	0.17	-	0.005	0.001
β_2	0.006	0.0001	-0.03	0.007
β_3	-0.06	0.004	-0.03	0.006
β_4	-	-	-0.09	0.04
	$R^2 = 0.99$		$R^2 = 0.48$	

Short run equation

$$\begin{aligned}\pi_{IH,t} = (1 - \beta_0)\bar{\pi}_{Q,t} + \beta_0 \pi_{Q,t} + \beta_1 T_{08q3,t} + \beta_2 \delta_{99Q4,t} + \beta_3 \delta_{08Q4,t} \\ + \beta_4 \left[p_{IH,t-1} - p^*_{IH,t-1}\right] + \epsilon_t\end{aligned} \tag{84}$$

Dynamic contributions See Figure 4.7.3.

Figure 4.7.3: Dynamic contributions, household investment deflator excluding VAT (in pp of growth rate)



4.7.4 Export deflator

The growth rate of the export deflator, $\pi_{X,t}$ is estimated as a one-step ECM. The estimation period is 2000Q1-2017Q4.

Target The parameter β_0 is calibrated to be consistent with IAD weights, i.e. with the import content of exports. For notations, see Table 4.7.7. Estimated and calibrated parameters are in Table 4.7.8. Foreign competitors' export price $p_{cx,t}$ is a weighted sum of trade partners' export prices.[54]

$$p^*_{X,t} = (1-\beta_2)\left[(1-\beta_0)p_{Q,t} + (\beta_0-\beta_1)p_{MNRJ,t} + \beta_1 p_{MO,t}\right] + \beta_2 p_{CX,t} \tag{85}$$

Short run equation

$$\pi_{X,t} = (1-\beta_0-\beta_1)\bar{\pi}_{Q,t} + \beta_0\pi_{Q,t} + \beta_1\pi_{M,t} + \beta_2\left[p_{X,t-1} - p^*_{X,t}\right] + \epsilon_t \tag{86}$$

Dynamic contributions The dynamics of the export price are mainly driven by the import price, see Figure 4.7.4.

[54]For more details about the computational aspects of this variables see Hubrich & Karlsson (2010).

Table 4.7.7: Variables used in section 4.7.4

Notation	Description
$\pi_{X,t}$	Export price inflation (in log)
$p^*_{X,t}$	Target of export price (in log)
$p_{X,t}$	Export price (in log)
$p_{Q,t}$	Value added price of market branches (in log)
$p_{MO,t}$	Import price (excluding energy) (in log)
$p_{MNRJ,t}$	Energy import price (in log)
$p_{CX,t}$	Foreign competitors' price (export) (in log)
$\bar{\pi}_{Q,t}$	Long run trend of the value added price inflation
$\pi_{M,t}$	Import price inflation (total)

Table 4.7.8: Estimates and calibrated parameters

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.33	-	0.89	0.12
β_1	0.28	0.02	0.48	0.03
β_2	0.27	0.07	-0.1	0.03

R^2 = 0.86 (One-step estimation)

4.7.5 Import deflators

We separately model the growth rate of the non-energy import price and the growth rate of the energy import price. These annual prices are then used to aggregate total imports in volume across both components. Having defined total imports in value as a simple sum of the two components, we compute the total import price as an accounting identity:

$$P_{M,t} = \frac{P_{MO,t}M_{O,t} + P_{MNRJ,t}M_{NRJ,t}}{M_t} \tag{87}$$

The decision to split the total import volume and price was motivated by the heterogeneity in the adjustment coefficient (for price) and the elasticity of substitution (for volume).

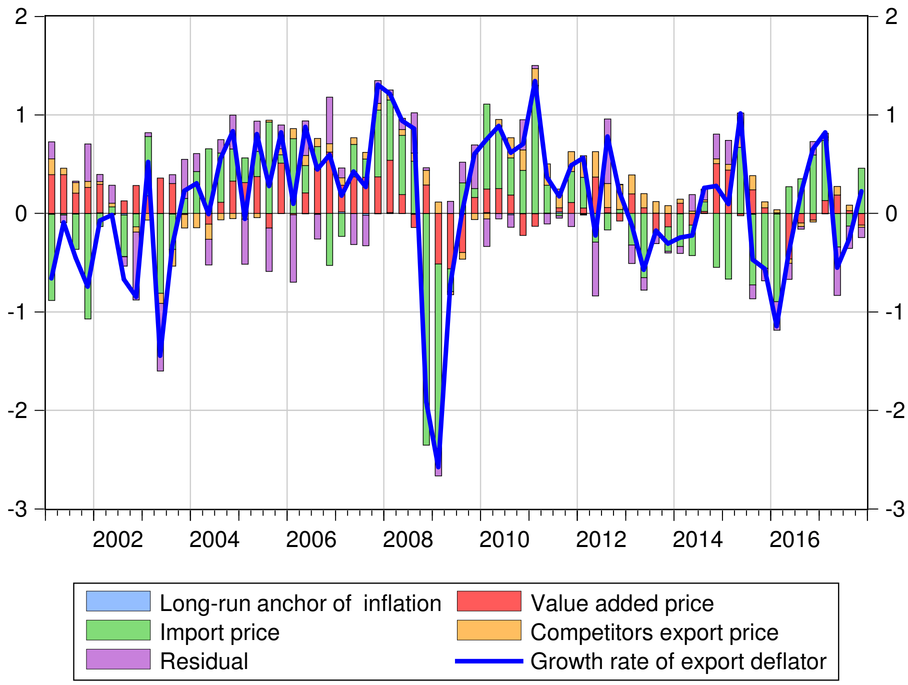
Non-energy import price The non-energy import price is estimated with a two-step ECM on the 2000Q1 - 2017Q4 sample.

Long run equation

$$p^*_{MO,t} = \beta_0 p_{Q,t} + (1 - \beta_0)p_{CM,t} + \beta_1\omega_t + \beta_2 \tag{88}$$

For notations see Table 4.7.9.

Figure 4.7.4: Dynamic contributions, export price (in pp of growth rate)



Short run equation

$$\begin{aligned}\pi_{MO,t} = (1-\beta_0-\beta_1)\bar{\pi}_t + \beta_0\pi_{CM,t} + \beta_1\pi_{MO,t-1} \\ +\beta_2\left[p_{MO,t-1} - p^*_{MO,t}\right] + \beta_3 + \epsilon_t\end{aligned} \tag{89}$$

Estimation results are presented in Table 4.7.10. Foreign competitors' price on import side $p_{cm,t}$ is a weighted sum of export prices with weights computed on import trade partners.[55]

Dynamic contributions The dynamics of the non-energy import price are mainly driven by the competitors' price (import), see Figure 4.7.5. An increasing weight of emerging countries is pushes $P_{MO,t-1}$ down over the whole sample period, while added value price has an upward effect on it.

Energy import price The energy import price is estimated with a one-step ECM on the 2000Q1 - 2017Q4 sample.

Long run equation

$$p^*_{MNRJ,t} = (1-\beta_0)p_{Q,t} + \beta_0(p_{OIL,t} - usd_t) + \beta_1 \tag{90}$$

For notations see Table 4.7.9.

[55]For more details about computational aspects of this variables see Hubrich & Karlsson (2010).

Table 4.7.9: Variables used in section 4.7.5

Notation	**Description**
$p^*_{MO,t}$	Target of non-energy import price (in log)
$p_{MO,t}$	Non-energy import price (in log)
$p_{Q,t}$	Value added price (in log)
$p^*_{MNRJ,t}$	Target of energy import price (in log)
$p_{MNRJ,t}$	Energy import price (in log)
$p_{OIL,t}$	Oil price (Brent, in log)
$p_{CM,t}$	Foreign competitors' price (import) (in log)
$\pi_{MO,t}$	Import price inflation (excluding energy)
$\bar{\pi}_{Q,t}$	Long run trend of the value added price inflation
$\pi_{M,t}$	Import price inflation (total)
ω_t	Weights of emerging countries (in log)
usd_t	Dollar/euro exchange rate (in log)

Table 4.7.10: Estimates and calibrated parameters, non-energy import price

	Long run		Short run	
	Coef.	s.e.	Coef.	s.e.
β_0	0.56	0.04	0.21	0.03
β_1	-0.31	0.01	0.36	0.08
β_2	0.06	0.002	-0.07	0.03
β_3	-	-	-0.002	0.0005
	$R^2 = 0.54$		$R^2 = 0.64$	

Short run equation

$$\pi_{MNRJ,t} = (1 - \beta_0)\bar{\pi}_t + \beta_0(\Delta p_{OIL,t} - \Delta usd_t) + \beta_1 \left[p_{MNRJ,t-1} - p^*_{MNRJ,t}\right] + \epsilon_t \tag{91}$$

Results are presented in Table 4.7.11.

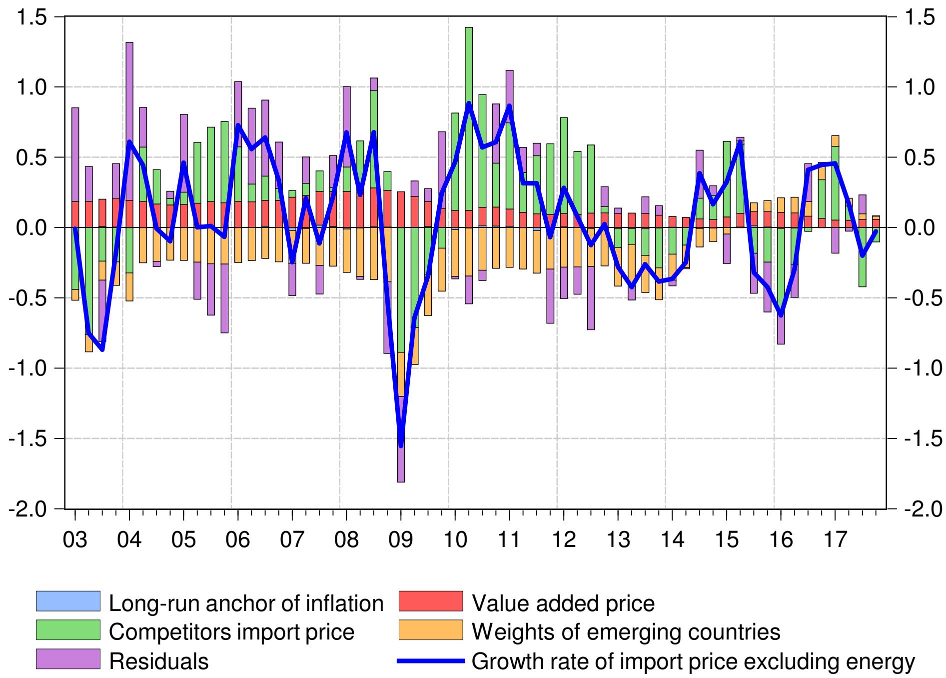
Dynamic contributions The dynamics of the energy import price are mainly driven by the euro-denominated oil price, see Figure 4.7.6.

4.7.6 Government consumption deflator

The main explanatory variable of the dynamics of the government consumption deflator ($P_{G,t}$) is the public sector salary. We have chosen to calibrate the elasticity of this deflator with respect to the efficient wage ($\pi_{W,t-1} - \Delta\bar{e}_{t-1}$) to be equal to the share of public sector wages (in value) in government spending (in value) in 2014Q4. This is estimated to be 0.54, see equation (92):

$$\pi_{G,t} = 0.54(\pi_{W,t-1} - \Delta\bar{e}_{t-1}) + (1 - 0.54)\bar{\pi}_{t-1} + \epsilon_t \tag{92}$$

Figure 4.7.5: Dynamic contributions, non-energy import price (in pp of growth rate)



4.8 Financial block

This section deals with the financial block: we cover, successively, the short and long government rates, the exchange rate and the determination of how the financial asset positions and asset income of the various sectors.

4.8.1 Short-term interest rate

The short rate i_t is measured by the 3-month Euribor. In simulation its dynamics are determined by a Taylor rule reacting to euro area inflation and the output gap, given by (93). Since in typical simulation applications these two variables are assumed exogenous, this equation in practice reduces to a simple $AR\,(1)$. In forecasting the short rate is fully exogenous.

$$(1-\lambda_i L)\,(i_t-\bar{\imath}_t)=(1-\lambda_i)\,(\alpha_i\,(\pi_{ea,t-1}-\bar{\pi}_{ea,t-1})+\beta_i\hat{y}_{ea,t-1}) \tag{93}$$

Note that (93) is simply the E-SAT core equation for the short rate; details relating to its estimation can be found in section 3.1.1.

Variation in the short rate is the primary driver of financial variables' dynamics in FR-BDF. Even though the short rate itself does not appear in any of the main behavioral equations, it determines the dynamics of the 10-year rate, which in turn either affects the real sector directly or is a key determinant of other interest rates (e.g. the user cost of capital for firms). Furthermore, as the short rate is a core component of the E-SAT expectations

Table 4.7.11: Estimates and calibrated parameters

	Long run Coef.	s.e.	Short run Coef.	s.e.
β_0	0.82	0.01	0.55	0.02
β_1	-3.5	0.05	-0.65	0.06
$R^2 = 0.92$ (1 step estimation)				

model, it has an effect on agents' behavior via all the expectation terms in the backward-looking setup.

As in the E-SAT model, the long-run anchor of the short-term interest rate, measured by the 5-year ahead forward rate of the 3-month Euribor rate, is simply modeled as an AR(1) process:

$$\left(1 - \tilde{\lambda}_{\bar{i}} L\right) \bar{\imath}_t = \epsilon_{\bar{i},t} \tag{94}$$

While we choose in E-SAT to calibrate the persistence of this variable to a value $\lambda_{\bar{i}} = 0.985$ in order to get term premia more in line with those assessed by models which take into account the possibility of zero lower bound, we do not impose such an assumption in the FR-BDF model. We set the persistence to its estimate equal to $\tilde{\lambda}_{\bar{i}} = 0.91$. Such an estimate is obtained using as a sample the pre-crisis period and it implies a fast normalization of interest rates in unconditional simulations.[56]

4.8.2 Long-term government interest rate

The long rate is measured by the return on 10-year French government bonds. Its dynamics are determined by the term structure equation (95) which relates the 10-year rate to an expectation component $\text{PV}\,(i)_{t|t-1}$ and the term spread $s_{10,t}$, which we assume to follow a simple $AR(1)$, as per (96). The estimation results are presented in Table 4.8.2. $\text{PV}\,(i)_{t|t-1}$ is determined using E-SAT; the relevant policy function is described in Table 4.8.3.[57] Figure 4.8.1 plots the elements of equation (95).

$$i_{10,t} = \text{PV}\,(i)_{t|t-1} + s_{10,t} \tag{95}$$

$$s_{10,t} = (1 - \rho_{10})\,\bar{s}_{10} + \rho_{10} s_{10,t-1} \tag{96}$$

The term structure equation (95) has its theoretical foundations in an approximation where the bond is modeled as having an infinite maturity with coupon payments that decay at a geometric rate. The calibration of the decay is chosen so that the distance between our approximated bond and the 10 year bond is minimized. The implied theoretical equation for $i_{10,t}$, the yield at maturity of a hypothetical long term bond, is then

$$i_{10,t} = (1 - \kappa_{10}) \sum_{s=0}^{\infty} \kappa_{10}^{s}\,(i_{t+s}) + s_{10,t} \tag{97}$$

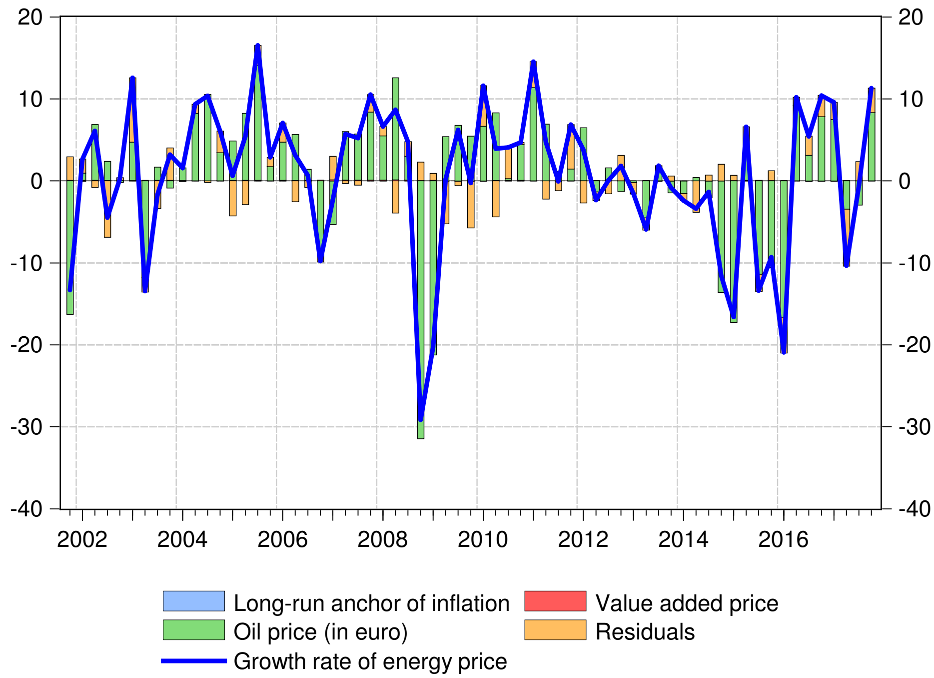
[56] In conditional forecasts, this choice has no influence, as this variable is simply extrapolated using technical assumptions of the forecast related to 5-year ahead futures.

[57] Note that no additional auxiliary equation is needed as i_t is included in the E-SAT core.

Table 4.8.1: Variables used in section 4.8

Notation	Description
i_t	3-month Euribor
$i_{10,t}$	Yield on 10-year French government bonds
$wacc_t$	Weighted average cost of capital
$i_{COE,t}$	Firm cost of equity
$i_{LB,t}$	Bank lending rate for firms
$i_{BBB,t}$	Yield on French BBB rated corporate bonds
$s_{10,t}$	Term spread between 10-year and 3-month French government bonds
$Y_{Fj,t}$	Net property income for agents of type j
$i_{Fj,t}$	Rate of return on financial assets for agents of type j
$\tau_{Tj,t}$	Financial transfers paid by agents of type j to households
$W_{j,t}$	Net stock of financial assets for agents of type j
$\tau_{Tj,t}$	Financial transfers paid by agents of type j to the households
$W_{j,t}$	Net stock of financial assets for agents of type j
$B_{j,t}$	Net financing need for agents of type j
$\bar{Y}_t$	Long run real output
$\bar{P}_{\bar{Y},t}$	Price of long run output
$Y_{IG,t}$	Net interest rate income, government
$Y_{IGP,t}$	Interest rate income paid, government
$Y_{IGR,t}$	Interest rate income received, government
$Y_{OG,t}$	Net other financial income, government
$Y_{OGP,t}$	Other financial income paid, government
$Y_{OGR,t}$	Other financial income received, government

Figure 4.7.6: Dynamic contributions, Energy import price (in pp of growth rate)



where κ_{10} is the decay – linked to the duration of the bond – and $s_{10,t}$ is the theoretical term spread.

Table 4.8.2: Coefficients and standard errors, term structure equation

Coef.	**Estimate**	**s.e.**
ρ_{10}	0.80	0.06
$\bar{s}_{10}$	0.15e-2	0.04e-2

The long rate plays an important role in FR-BDF as the foundation on which the construction of the rates paid by the private sector are based, particularly the user cost of capital $r_{KB,t}$ and the household bank lending rate $i_{LB,t}$. Furthermore, it is used to determine the income received by the various agents on their financial assets.

4.8.3 Private interest rates

Weighted average cost of capital (WACC) The weighted average cost of capital $wacc_t$ – a key component of the user cost of capital – is determined as a weighted average of the main components of funding used by French firms: Cost of Equity (COE) $i_{COE,t}$, the firm bank lending rate $i_{LB,t}$ and a bond rate $i_{BBB,t}$ which is represented by the rate on BBB-rated corporate bonds, as computed by Merrill Lynch-Bank of America. The weights on the

Figure 4.8.1: Long-term government interest rate and its components, annualized percent

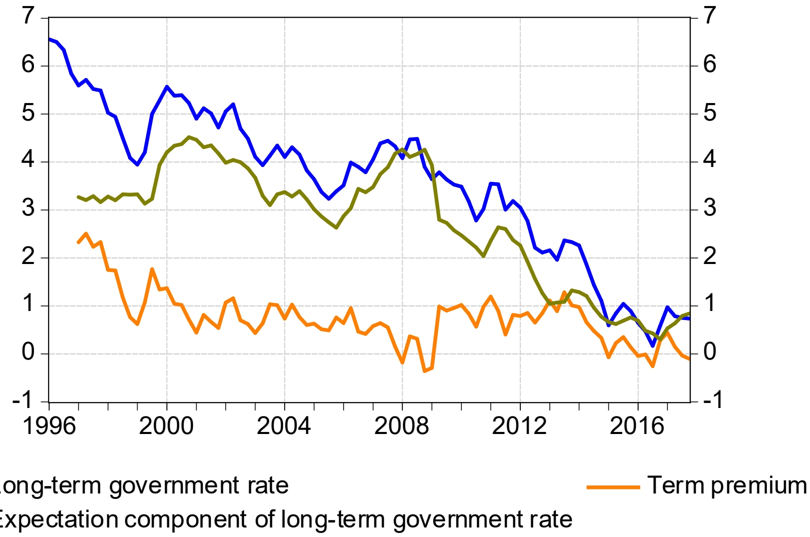


Table 4.8.3: Coefficients of the policy function for the expectation of the 3-month bond rate

VAR model	**Coefficient**
Constant	0.003
$\hat{y}_{t-1}$	0
i_{t-1}	0.16
$\bar{i}$	0.51
$\pi_{t-1} - \bar{\pi}$	0
$\hat{y}_{EA,t-1}$	0.026
$\pi_{EA,t-1} - \bar{\pi}_{EA,t-1}$	0.041

components have been computed as historical averages of their shares in the liabilities of French corporations. See Carluccio et al. (2019) for details. Figure 4.8.2 plots the WACC and its components.

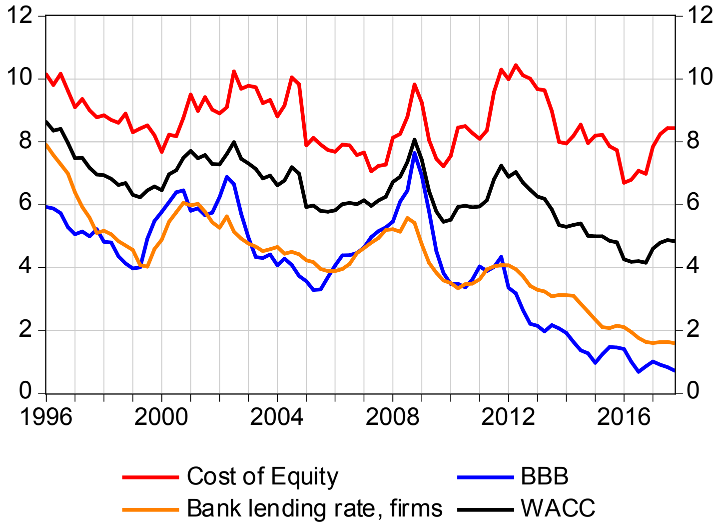
$$wacc_t = 0.5 i_{COE,t} + 0.3 i_{LB,t} + 0.2 i_{BBB,t} \tag{98}$$

In simulation all the components of the WACC are determined as the sum of a spread term $s_{j,t}$ with $j \in \{COE, LB, BBB\}$ and the long rate:

$$i_{j,t} = s_{j,t} + i_{10,t} \tag{99}$$

The three spreads are all assumed to follow simple AR(1) processes of the form (100) and are plotted in Figure 4.8.3. The corresponding estimated coefficients are reported in Table 4.8.4.

Figure 4.8.2: Weighted average cost of capital and its components, annualized percent



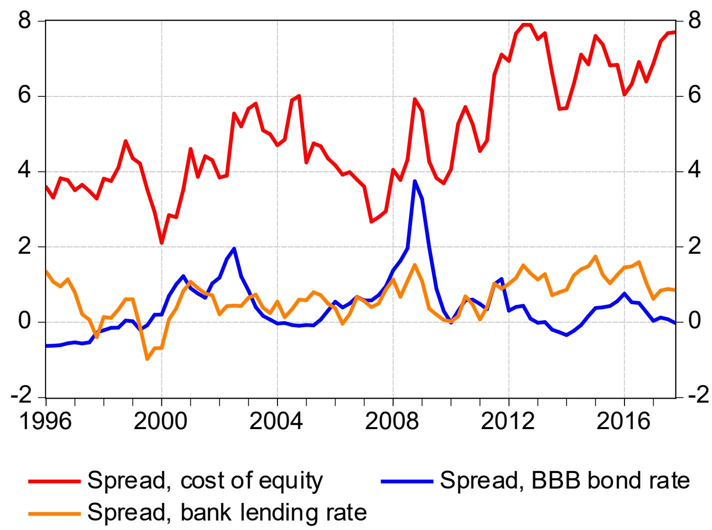
$$s_{j,t} = \bar{s}_j (1 - \rho_j) + \rho_j s_{j,t-1} \tag{100}$$

Table 4.8.4: Coefficients and standard errors, spread equations

Coef.	Estimate	s.e.
ρ_{COE}	0.92	0.05
$\bar{s}_{COE}$	1.40e-2	0.2e-2
ρ_{LB}	0.77	0.05
$\bar{s}_{LB}$	0.30e-2	0.05e-2
ρ_{BBB}	0.94	0.026
$\bar{s}_{BBB}$	0.02e-2	0.1e-2

Measure of cost of equity The observed series for $i_{COE,t}$ used in the estimation have been computed using the methodology of Carluccio et al. (2019). The computation is based on an extension of the standard dividend-discount model in the style of Fuller & Hsia (1984), which is used to compute the risk premium $R_{m,t}$ for French firms. Combining this with separately estimated sensitivities to risk - i.e. Capital Asset Pricing Model (CAPM) betas - makes it possible to construct observed $i_{COE,t}$.

Figure 4.8.3: Spreads between various rates and the 10 year government rate, annualized percent



4.8.4 Exchange rates

There are two exchange rates in FR-BDF: the effective exchange rate of the euro area and the exchange rate between the dollar and the euro.[58] Both are expressed in direct quotes and modeled using an uncovered interest rate parity (UIP) condition, which means that exchange rates are defined as functions of the countries' interest rate differentials:

$$\frac{\Xi_{t+1}}{\Xi_t} = \frac{(1 + i_{F,t})}{(1 + i_t)} \tag{101}$$

where Ξ_t stands for the direct exchange rate (i.e. units of foreign currency per unit of domestic currency), i_t and $i_{F,t}$ are domestic and foreign short run interest rates, respectively. The UIP condition (101) requires that each asset must have the same nominal return and this interdependence is ensured through capital movements between two countries such that the preceding no-arbitrage condition holds.

We define the log real exchange rate q_t:

$$q_t \equiv \xi_t + p_{EA,t} - p_{F,t} \tag{102}$$

where p_t and p_t^F are respectively the log of euro area and foreign prices, we can rewrite the UIP condition for the real exchange rate, in terms of interest rate and inflation rate differentials, in logs:

$$\begin{aligned} q_t - (p_{EA,t} - p_{F,t}) &= q_{t+1} - (p_{EA,t+1} - p_{F,t+1}) + (i_t - i_{F,t}) \\ q_t &= q_{t+1} + (i_t - i_{F,t}) - (\pi_{EA,t+1} - \pi_{F,t+1}) \end{aligned} \tag{103}$$

[58] The effective exchange rate of the euro area measures the value of the euro with respect to a bundle of 38 countries.

Solving forward equation (103) and using the definition of the log real exchange rate (102), we obtain equation (104) that we use as a benchmark for the empirical specification of the two nominal exchange rates.

$$\xi_t + p_t^{EA} - p_t^F = \sum_{\kappa=0}^{\infty}\Big(i_{t+\kappa} - i_{F,t+\kappa}\Big) - \sum_{\kappa=0}^{\infty}\Big(\pi_{EA,t+1+\kappa} - \pi_{F,t+1+\kappa}\Big) \tag{104}$$

Table 4.8.5: Variables used in section 4.8.4

Notation	Description
$\xi_{EA,t}$	Effective exchange rate of the euro area, in log
$\xi_{\$,t}$	Dollar/Euro exchange rate, in log
$i_t/i_{F,t}$	Domestic/foreign short run interest rate
$p_{EA,t}/p_{F,t}$	Euro area/foreign GDP deflator, in log
$\pi_{EA,t}/\pi_{F,t}$	Euro area/foreign quarterly inflation rate (GDP deflator
$\xi_{FR,X,t}/\xi_{FR,M,t}$	French exchange rate on export/import side
$P_{cm,t}/P_{cx,t}$	Foreign competitors' price (import/export)
$\omega_{cm,t}^{in}/\omega_{cx,t}^{in}$	Share of intra-EA trade flows in French imports and exports
$P_{cm,t}^{in}/P_{cx,t}^{in}$	Intra-EA foreign competitors' price (import/export)
$P_{cm,t}^{ex}/P_{cx,t}^{ex}$	Extra-EA foreign competitors' price (import/export)
$P_{cm,fc,t}^{ex}/P_{cx,fc,t}^{ex}$	Extra-EA foreign competitors' price expressed in foreign currency (import/export)
E-SAT variables	See Table 4.2.1

Both nominal exchange rates $\xi_{EA,t}$ and $\xi_{\$,t}$ are estimated with a constant and an AR(1) process in residuals estimated on the sample 1999Q2-2010Q1 before the sovereign debt crisis. For notations see Table 4.8.5.

The infinite sums of the short run interest rates and inflation rates are computed as non-discounted present values (PV_{nd}) by inverting the corresponding models: the E-SAT model in the case of i_t and $\pi_{EA,t+1}$ and AR(1) models for $i_{F,t}$ and $\pi_{F,t+1}$ (see equations (107) and (109)). The exchange rates of the euro vis-a-vis the dollar and the bundle of 38 other currencies ($\xi_{\$,t}$ and $\xi_{F,t}$ respectively) are estimated in the following forms, first for the Dollar/Euro exchange rate:

$$\begin{aligned}\xi_{\$,t} + p_t^{EA} - p_t^F \;&=\; \beta + \Big[\big(PV_{nd}(i)_{t|t-1} - PV_{nd}(i_F)_{t|t-1}\big) \\ &\quad -\big(PV_{nd}(\pi_{EA})_{t+1|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\big)\Big] + \eta_t\end{aligned}$$

with $(1-\rho L)\eta_t = \epsilon_t$, which can be rewritten as follows:

$$\begin{aligned}(1-\rho L)(\xi_{\$,t} + p_t^{EA} - p_t^F) \;&=\; (1-\rho)\beta + (1-\rho L)\Big[\big(PV_{nd}(i)_{t|t-1} - PV_{nd}(i_F)_{t|t-1}\big) \\ &\quad -\big(PV_{nd}(\pi_{EA})_{t+1|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\big)\Big] + \epsilon_t\end{aligned} \tag{105}$$

And for the Euro effective exchange rate:

$$\begin{aligned}\xi_{EA,t} + p_t^{EA} - p_t^F &= \beta + \Big[\big(PV_{nd}(i)_{t|t-1} - PV_{nd}(i_F)_{t|t-1}\big) \\ &\quad -\big(PV_{nd}(\pi_{EA})_{t+1|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\big)\Big] + \eta_t\end{aligned}$$

with $(1-\rho L)\eta_t = \epsilon_t$ on, which can be rewritten as follows:

$$\begin{aligned}(1-\rho L)(\xi_{EA,t} + p_t^{EA} - p_t^F) &= (1-\rho)\beta + (1-\rho L)\Big[\big(PV_{nd}(i)_{t|t-1} - PV_{nd}(i_F)_{t|t-1}\big) \\ &\quad -\big(PV_{nd}(\pi_{EA})_{t+1|t-1} - PV_{nd}(\pi_F)_{t+1|t-1}\big)\Big] + \epsilon_t \qquad (106)\end{aligned}$$

Estimated coefficients are presented in Table 4.8.6. Policy functions of the present value of non-discounted sums of future domestic short run interest rates ($PV_{nd}(i)_{t|t-1}$) and of future domestic inflation rate ($PV_{nd}(\pi_{EA})_{t+1|t-1}$) are given in Table 4.8.7. It is interesting to note that the semi-elasticity of $PV_{nd}(i)_{t|t-1}$ with respect to $(i_{t-1} - \bar{i})$ is defined largely by the euro area variables. If one considers the E-SAT model without euro area variables (leaving only three equations: French IS and Phillips curves and an AR(1) instead of Taylor rule) and inverts it, then this elasticity becomes 12.2 instead of 3.14.[59]

Table 4.8.6: Coefficients and standard errors of exchange rate equations

	$\xi_{\$,t}$ eq. 105 Coef.	s.e.	$\xi_{EA,t}$ eq. 106 Coef.	s.e.	$i_{F,t}$ eq. 107 Coef.	$PV_{nd}(i_F)$ eq. 108 Coef.	$\pi_{F,t}$ eq. 109 Coef.	s.e.	$PV_{nd}(\pi_F)$ eq. 110 Coef.
β	0.15	0.23	0.04	0.15	-	-	-	-	-
γ	-	-	-	-	-	-	-	-	-
ρ	0.95	0.03	0.94	0.04	0.96	0.96	0.47	0.09	0.47

In order to construct $PV_{nd}(i_F)_{t|t-1}$ we need a model for foreign short run interest rates. To proxy the latter, we apply the Federal Reserve's 3-month interest rate modeled as an AR(1) process with mean reversion:[60]

$$i_{F,t} = \rho i_{F,t-1} + (1-\rho)\bar{i} + \epsilon_t \qquad (107)$$

This implies the following present value of non-discounted sums of future foreign short run interest rates:

$$PV_{nd}(i_F)_{t|t-1} = \sum_{\kappa=0}^{\infty}(i_{F,t+\kappa} - \bar{i}) = \frac{\rho}{(1-\rho)}(i_{F,t-1} - \bar{i}) \qquad (108)$$

For the sake of simplicity, ρ in equation (108) is set to be the same as that of the French short run interest rate.[61] We apply the method to construct the $PV_{nd}(\pi_F)_{t+1|t-1}$. We proxy the

[59] 12.2 is obtained by computing $\frac{\lambda_i}{1-\lambda_i} = \frac{0.9243}{(1-0.9243)}$, where λ_i is the persistence parameter of the short run interest rate, see Table 3.1.1.

[60] This avoids modeling the US economy, which is assumed to be exogenous.

[61] Such a simple relation as presented in (108) is not sufficient to recover the true value of this parameter.

foreign price level and inflation rate using the US GDP deflator and estimate the following AR(1) process:

$$\pi_{F,t} = \rho\pi_{F,t-1} + (1-\rho)\bar{\pi} + \epsilon_t \tag{109}$$

where $\bar{\pi}$ is the steady-state inflation rate.[62] We finally obtain:

$$PV_{nd}(\pi_F)_{t+1|t-1} = \sum_{\kappa=0}^{\infty}(\pi_{F,t+1+\kappa} - \bar{\pi}) = \frac{\rho}{(1-\rho)}(\pi_{F,t-1} - \bar{\pi}) \tag{110}$$

Table 4.8.7: Policy function of the expected sum of the future short run interest rate

VAR Model variables	**Policy function $PV_{nd}(i)$**	**Policy function $PV_{nd}(\pi_{EA})$**
$i_{t-1} - \bar{i}$	3.14	-3.15
$\bar{i}_{t-1} - \bar{i}$	62.52	3.15
$\hat{Q}_{EA,t-1}$	1.26	0.40
$\pi_{EA,t-1} - \bar{\pi}$	1.64	0.12
$\bar{\pi}_{EA,t-1} - \bar{\pi}$	-1.64	13.11
$\hat{y}_{t-1}$	-	-
$\pi_{Q,t-1} - \bar{\pi}_{Q,t-1}$	-	-

Foreign competitors' prices of exports and imports expressed in euros ($P_{cx,t}$ and $P_{cm,t}$ respectively) are modeled as weighted average of intra- and extra-EA foreign competitors prices. Weights of competitors on the import side are computed as simple shares of intra and extra imports in French imports. On the export side, these weights are theselve computed with weighted sums of export shares, in order to take into account thrird-market effects (see Hubrich & Karlsson 2010). We denote shares of *intra*-EA exports and *intra*-EA imports respectively by $\omega^{in}_{x,t}$ and $\omega^{in}_{m,t}$.[63]

$$P_{cx,t} = \omega^{in}_{x,t} \times P^{in}_{cx,t} + (1-\omega^{in}_{x,t}) \times P^{ex}_{cx,t} \tag{111}$$

$$P_{cm,t} = \omega^{in}_{m,t} \times P^{in}_{cm,t} + (1-\omega^{in}_{m,t}) \times P^{ex}_{cm,t} \tag{112}$$

Intra-EA foreign competitors prices for exports and imports (respectively $P^{in}_{cx,t}$ et $P^{in}_{cm,t}$) are assumed exogenous, but further developments of FR-BDF may lead to link them to Euro Area variables.

Extra-EA foreign competitors prices for exports and imports in euros (respectively, $P^{ex}_{cx,t}$ et $P^{ex}_{cm,t}$) vary with the effective exchange rate of the euro area Ξ_{EA} and with foreign competitors' prices of exports and imports expressed in foreign currencies (respectively, $P_{cx,fc,t}$ and $P_{cm,fc,t}$), which are exogenous:

$$P^{ex}_{cx,t} = \frac{P^{ex}_{cx,fc,t}}{\Xi_{EA,t}} \tag{113}$$

[62] We therefore make the simplyfing assumption that the steady-state inflation rate is equal in the US and the Euro Area.

[63] In the historical sample, the weights are observed at annual frequency and then interpolated at quarterly frequency. In simulation, we assume they are constant and prolong them at their last value.

$$P^{ex}_{cm,t} = \frac{P^{ex}_{cm,fc,t}}{\Xi_{EA,t}} \tag{114}$$

Oil prices in euros are modeled as a product of oil prices in dollars and the dollar/euro exchange rate:

$$P_{oil,t} = \frac{P_{oil,\$,t}}{\Xi_{\$,t}} \tag{115}$$

4.8.5 Net property income and net asset positions

The framework for determining the net property income $Y_{Fj,t}$ of the various agents $j \in \{\text{Firms, Government, Households, Non-profit organizations}\}$ of the model is based on

$$Y_{Fj,t} = i_{Fj,t} W_{j,t-1} \tag{116}$$

where $i_{Fj,t}$ is the agent-specific rate of return on the net stock of financial wealth $W_{j,t-1}$ of agent j.[64] We denote the four agents with the subscripts F, G, H and N for brevity. For most agents we deviate from this baseline. More specifically, we assume that for firms

$$Y_{FF,t} = i_{FF,t} W_{F,t-1} - \tau_{TF,t} \bar{Y}_t \bar{P}_{\bar{Y},t} \tag{117}$$

where the term $\tau_{TF,t}\bar{Y}_t$ represents the real financial transfers made by the firms to households in order to stabilize their net asset ratio. Similarly, the financial income of the non-profit organizations is modified to account for transfers $\tau_{TN,t}\bar{Y}_t$ between them and households:

$$Y_{FN,t} = i_{FN,t} W_{N,t-1} - \tau_{FN,t} \bar{Y}_t \bar{P}_{\bar{Y},t} \tag{118}$$

The households' financial income is then

$$Y_{FH,t} = i_{FH,t} W_{H,t-1} + \tau_{TF,t} \bar{Y}_t \bar{P}_{\bar{Y},t} + \tau_{FN,t} \bar{Y}_t \bar{P}_{\bar{Y},t} \tag{119}$$

i.e. they receive payments directly from their stock of wealth $W_{H,t-1}$ and as transfers from firms and non-profit organizations. Note that $\bar{Y}_t$ is long run real output and $\bar{P}_{\bar{Y},t}$ is its price – the product is then nominal long run output.

The stocks of financial wealth $W_{j,t}$ are assumed to evolve following

$$\Delta W_{j,t} = B_{j,t} \tag{120}$$

where $B_{j,t}$ is the net financing capacity of agents of type j, with the exception of firms, for whom we also model the revaluations $v_{F,t}$ of their asset stock so that

$$\Delta W_{F,t} = B_{F,t} + v_{F,t} \tag{121}$$

$$v_{F,t} = \gamma \bar{Y}_t \bar{P}_{\bar{Y},t} \tag{122}$$

with $\gamma = -0.018$, which is the mean of the ratio of reevaluations to $\bar{Y}_t \bar{P}_{\bar{Y},t}$.

[64] Data about net financial wealth comes from financial accounts.

The transfer policies of firms and non-profit organizations are given by

$$\begin{aligned}\tau_{TF,t} &= (1-\rho_{stab,1})\,\tau_{TF,t-1}+\rho_{stab,1}\tau_{TF}^{*} \\ &\quad -\rho_{stab,2}\left(\frac{-B_{F,t}+\gamma\bar{Y}_t\bar{P}_{\bar{Y}_t}}{Y_tP_{Y,t}}+\frac{W_F}{Y}\frac{\exp\left(g+\bar{\pi}\right)-1}{\exp\left(g+\bar{\pi}\right)}\right)\end{aligned} \tag{123}$$

$$\tau_{TN,t}=(1-\rho_{stab,1})\,\tau_{TN,t-1}+\rho_{stab,1}\tau_{TN}^{*}-\rho_{stab,2}\left(\frac{-B_{N,t}}{Y_tP_{Y,t}}+\frac{W_N}{Y}\frac{\exp\left(g+\bar{\pi}\right)-1}{\exp\left(g+\bar{\pi}\right)}\right) \tag{124}$$

where $\rho_{stab,1}=0.1$ and $\rho_{stab,2}=0.1$ are calibrated. $\tau_{TF}^{*}=0.026$ and $\tau_{TN}^{*}=0.00026$ are exogenous long run targets – constructed using a simulation method – for the rate at which dividends and transfers are paid to households. $\frac{W_F}{Y}=-0.7*4$ and $\frac{W_N}{Y}=0.02*4$ are exogenous, calibrated targets for the ratio of assets to nominal output for firms and non-profit organizations, respectively. Note that these two rules are strongly based on a similar policy rule used by the government to ensure that its net financial asset-to-GDP ratio is stable in the long run:

$$\tau_{TG,t}=(1-\rho_{stab,1})\,\tau_{TG,t-1}+\rho_{stab,1}\tau_{TG}^{*}-\rho_{stab,2}\left(\frac{-B_{G,t}}{Y_tP_{Y,t}}+\frac{W_G}{Y}\frac{\exp\left(g+\bar{\pi}\right)-1}{\exp\left(g+\bar{\pi}\right)}\right) \tag{125}$$

where $\frac{W_G}{Y}=-0.4*4$ and $\tau_{TG}^{*}=0.16$ are constructed similarly to their counterparts in equations (123) and (124). The key difference is that this rule is not used to regulate financial transfers, but to determine the share of social transfers excluding unemployment benefits and social transfers in kind relative to nominal GDP. See section 4.10 for details.

The rates of return $i_{j,t}$ are assumed to follow for all agents j an autoregressive distributed lag process

$$i_{j,t}=\rho_{j,0}\left(1-\rho_{j,1}\right)+\left(1-\rho_{j,1}\right)i_{10,t}+\rho_{j,1}i_{j,t-1} \tag{126}$$

the coefficients of which are shown in Table 4.8.8. We assume that for all types of agents $\rho_{j,1}=0.983$ – calibrated so that the process has a half-life of 40 quarters – and that $\rho_{G,0}=0$ to ensure convergence to $i_{10,t}$. The long run premia are estimated from data as the historical means of the ratio of total asset income to total asset stock minus the mean of $i_{10,t}$.

Table 4.8.8: Coefficients and standard errors, asset return processes

Coef.	**Estimate**
$\rho_{F,0}$	-0.0037
$\rho_{G,0}$	0
$\rho_{H,0}$	-0.0007
$\rho_{N,0}$	-0.001

Finally, due to the requirements of accounting appropriately for government finances, we model separately the interest payments made by the public sector - $Y_{IGP,t}$ - and asset income payments excluding interest income made by the public sector, i.e. $Y_{OGP,t}$, where "O" stands for "other". First, the interest payments are computed as

$$Y_{IGP,t}=Y_{IGR,t}-Y_{IG,t}$$

i.e. as the difference between interest payments received $Y_{IGR,t}$ and net interest income $Y_{IG,t}$. We assume that interest payments received $Y_{IGR,t}$ are a constant share - 0.011% - of long run output $\bar{Y}_t$, while the net interest income is computed as

$$Y_{IG,t} = Y_{FG,t} - Y_{OG,t}$$

i.e. as the difference between total net asset income $Y_{FG,t}$ (described above) and net asset income excluding interest payments $Y_{OG,t}$. This, in turn is simply the difference between receipts and payments, i.e.

$$Y_{OG,t} = Y_{OGR,t} - Y_{OGP,t}$$

both of which are assumed to be constant shares of $\bar{Y}_t$ - 0.006 and 0.05 percent, respectively.

4.9 Trends

FR-BDF includes several trend variables denoted by $\bar{x}_t$ for any variable x_t.[65] These trends are introduced for two main reasons. First, trend variables are often needed within the PAC framework to construct expectations for target variables x_t^*. For instance, we quite often decompose expectations of *target* variables between the trend growth rate of the target $(PV(\Delta\bar{x}^*)_{t|t-1})$ and the change in the gap between the target and its trend $(PV(\Delta\hat{x}^*)_{t|t-1})$, as in section 4.5.2. Second, we need several trend variables to evaluate long run output. For instance, we use trends of labor force, hours worked per employee or non-market branches GDP.

In the historical sample and for the purpose of estimation, these trend variables are obtained by applying the HP filter on quarterly data with $\lambda_{HP} = 1600$. Then, in conditional forecasting and unconditional simulation, trend variables are projected using single exponential smoothers. For any variable x_t and its trend $\bar{x}_t$, we use the following single exponential smoother:

$$\bar{x}_t = \rho_{HT}(\bar{x}_{t-1} + g_x) + (1 - \rho_{HT})x_t + \varepsilon_t \tag{127}$$

where g_x is the long run growth rate of the variable; for stationary variables, this term naturally cancels out.

For some variables, we observed highly persistent residuals ε_t which had undesirable properties in simulation when residuals are set to zero.[66] Consequently, for these variables, we use a modified single exponential smoother with AR(1) residuals:

$$\begin{aligned} \bar{x}_t &= \rho_{HT}(\bar{x}_{t-1} + g_x) + (1 - \rho_{HT})x_t + \eta_t \\ \eta_t &= \rho_{TVP}\eta_{t-1} + \varepsilon_t \end{aligned}$$

which we express using a nonlinear expression:

$$\bar{x}_t - \rho_{TVP}\bar{x}_{t-1} = \rho_{HT}\big(\bar{x}_{t-1} - \rho_{TVP}\bar{x}_{t-2} + (1-\rho_{TVP})g_x\big) + (1-\rho_{HT})\big(x_t - \rho_{TVP}x_{t-1}\big) + \varepsilon_t \tag{128}$$

[65] These filtered variables are referred to as "trend" variables as they capture low-frequency evolutions. In some cases, these variables can, however,be stationary.

[66] In practice, variables for which we observe large and persistent residuals often appear to be nonstationary.

Both ρ_{HT} and ρ_{TVP} are calibrated in an *ad hoc* fashion. We set $\rho_{HT} = 0.95$ to achieve around 90% of convergence toward the observed level x_t within 40 quarters and $\rho_{TVP} = 0.83$ such that only 10% of the initial residuals persist at the end of the usual forecasting horizon (12 quarters).

Equations (127) and (128) are the general framework for modeling trends within FR-BDF. However, in practice, we can slightly deviate from these equations in order to preserve the model's dynamic properties. As any smoother algorithm, the exponential smoother uses the actual variable to evaluate its trend. In turn, in the PAC framework, the trend of the target and the gap between the target and its trend are included in expectation variables. This creates a loop between endogenous variables (e.g. the target of employment) and their trends, which can have undesirable properties (e.g. amplifying or creating oscillatory dynamics) in conditional forecasting and unconditional simulation exercizes.

From what precedes, we conclude that trends should be exogenous when feasible. Thus, we choose to anchor the exponential smoother not to the *actual* variable that we wish to smooth, but rather to the long run equilibrium of the variable. For instance, in the employment equation presented in section 4.5.2, the trend of the employment target $\bar{n}^*_{S,t}$ is anchored to the long run equilibrium level of salaried employment of market branches $\widetilde{n^*_{S,t}}$, defined by:

$$\widetilde{n^*_{S,t}} = \log\left((1 - u_{N,t})\bar{\nu}\bar{\psi}_t\overline{POP}_t\right) \tag{129}$$

where $\bar{\nu}$ is the historical average of the ratio of salaried employment to total employment of market branches, $\bar{\psi}_t$ is the trend share of market branches employment in total employment, $\overline{POP}_t$ is the trend of the labor force and $u_{N,t}$ is the long run equilibrium rate of unemployment.

Another example is the trend of the marginal return on capital services $\overline{Q'_{K,t}}$ presented in section 4.3. In this case, we anchor the exponential smoother not to the observed marginal return but rather to the steady state of the capital demand equation (26). We define $\widetilde{Q'_{K,t}}$ as follows:

$$\widetilde{Q'_{K,t}} \equiv \mu\frac{\overline{\tilde{r}_{K,t}}}{P_{Q,t}} \tag{130}$$

where $\overline{\tilde{r}_{K,t}}$ is evaluated using the long run anchors of WACC, inflation expectations, a constant depreciation rate of capital services $\tilde{\delta}$. In particular, we use the *observed* relative price of investment with respect to the value added price of market branches, because we are not able to obtain an analytical solution for it. As a result, $\widetilde{Q'_{K,t}}$ is not fully but merely quasi-exogenous.

Four trend variables have a particular status and depart from the modelling framework that we have presented here: $\bar{i}_t$ the long run trend of the short run interest rate, $\bar{\pi}_t$ the long run trend of the value added price inflation of market branches, $\bar{\pi}_{EA,t}$ the long run trend of euro area GDP deflator inflation and $\bar{E}_t$ the trend labor efficiency. See section 3.1.2 for details about the first three variables. Finally, trend labor efficiency follows a deterministic trend, as described in section 4.3.

4.10 Accounting framework and public finances

In our accounting framework we decompose production and labor market variables into market and non-market branches.[67] As the accounting framework is based on quarterly national accounts, some choices were made according to data availability in these accounts. For example, as the volume of value added is available only in branch accounts and not in sector accounts, we include in the accounting framework branch accounts in value and in chained price volume, but only sector accounts in value.[68] The branch account decomposition also plays an important role in the interaction with the HICP model used for inflation forecasting, known as MAPI (Model for Analysis and Projection of Inflation in France, see de Charsonville et al., 2017, for a detailed presentation).

The second important decomposition is based on sector accounts. In this dimension, we distinguish five economic agents:

- firms, which correspond to non-financial and financial corporations;
- government, which corresponds to general government;
- households, which correspond to households including unincorporated enterprises;
- non-profit organizations, which correspond to non profit institutions serving households (NPISH);
- rest of the world.

The model includes a detailed account in value for each agent (from its value added to its borrowing capacity). For firms, we group together non-financial and financial corporations, because such a split is generally unnecessary for participating in the Eurosystem's forecasting exercises (except for some credit variables).[69] For households, we include unincorporated enterprises, because the headline variables needed for forecasting, i.e. the savings ratio and the gross disposable income, should concern households including those agents.[70] Although their weight in GDP is very small, we include an account of non-profit organizations in order to cover all sectors of national accounts.[71] Finally, the bottom lines of the accounts of the government and rest of the world determine two important headline variables, namely the government balance and the current account. The variables of the rest of the world are generally simply determined by the variables of other agents through accounting identities.

Although this sector decomposition is generally used in FR-BDF, we make an exception for business investment: as investment decisions of unincorporated enterprises are closer to

[67] As indicated earlier in the section about he supply block, market branches correspond to branches AZ to MN in Insee codes, while non-market branches correspond to OQ.

[68] For chained price aggregation, as in quarterly national accounts provided by Insee or Eurostat, our model uses annual overlap formulas.

[69] For details about the Eurosystem's staff macroeconomic projection procedures, see the guide provided here.

[70] Moreover, quarterly national accounts do not contain a full account of households excluding unincorporated enterprises.

[71] Another possibility would have been to include this NPISH sector within the field of households but this would have not complied with the need to forecast the saving rate of households only.

those of corporate firms than to those of households, we estimate behavioral equations for business investment including unincorporated enterprises on one side and household investment excluding these unincorporated enterprises on the other. In addition, bridge equations are used to ensure consistent dynamics of corporate firms' and households' investments with these equations.

The government block is particularly detailed and designed in such a way as to allow interactions with the public finance model of the Banque de France, known as MAPU (*Maquette Agrégée des finances PUbliques*). In particular, we have created two modes for many variables in this block:

- A forecasting mode, where many nominal variables are exogenized and come directly from the public finance model;
- A simulation mode, where these variables are endogenous and depend on exogenous effective tax rates or exogenous ratios.

More precisely, in the simulation mode, we adopt the following common principles on receipt and spending sides:

- On the receipt side, each receipt is determined by an exogenous effective tax rate and on an endogenous tax basis;
- On the spending side, some spending variables are directly related to macroeconomic aggregates with effective rates. For example, unemployment benefits are directly related to unemployment and wage per capita. For other spending variables, such as intermediate consumption or government investment, the ratio of their volume relative to long run output is assumed to be exogenous.

In the simulation mode, one exception to these principles concerns social transfers excluding unemployment benefits and social transfers in kind $T_{G,t}$. As is the case for many variables on the spending side, their volume is related to long run output through a ratio, $\tau_{TG,t}$, but this ratio is endogenized in this mode with equation (125) to ensure the convergence of the government's net asset ratio toward its long run target (see section 4.8.5).

Finally, we take into account two peculiarities of the government account compared to other sector accounts. First, for this agent, the operational surplus is not determined as its value added net of paid compensations, but is assumed to be determined as an exogenous share of long run output. Indeed, this surplus is mainly its consumption of fixed capital and, to simplify, we assume the share of this consumption in long run GDP to be stable. Once the operating surplus is determined, the value added of government is simply determined as the sum of this surplus, paid compensations and other taxes on production (net of corresponding subsidies). Second, another peculiarity of the government account concerns final consumption, which is an accounting construction: in fact, the government does not consume public services, while households and firms do; however, this consumption cannot be attributed to households or to firms since there are no observable monetary transactions; hence, national accountants attribute it to the general government itself. For these reasons, government consumption is determined as the sum of government value added, government intermediate consumption and paid social benefits in kind, net of government sales of services.

4.11 Model changes for conditional projections within BMPE exercises

In conditional forecasts, FR-BDF is modified to comply with the Eurosystem's Broad Macroeconomic Projection Exercises (BMPE) guidelines (ECB, 2016). Table 4.11.1 summarizes model changes for conditional forecasts.

First, some exogenous variables of FR-BDF – which are projected at constant growth rates (e.g. world demand, competitors' prices) – are projected using the Eurosystem's assumptions in conditional forecast exercises.

Second, several endogenous variables are taken from the Eurosystem's assumptions determined within the BMPE, particularly variables in the financial block: both expected term structure and Uncovered Interest Parity (UIP) equations no longer determine the long-term nominal rate, the nominal effective exchange rate and dollar/euro nominal exchange rate. As a result, crude oil prices in euro and competitors' prices are also exogenized, following the Eurosystem's assumptions.[72] The Eurosystem's assumptions are determined and revised several times within an exercise. More importantly, these assumptions account for external spillovers, using intermediate projections from National Central Banks (NCBs) within the Eurosystem.

Third, three auxiliary models are used to compute (i) bank lending rates (BLR) to firms and households, (ii) HICP forecasts, (iii) public finances variables. BLR models used within the Eurosystem NCBs must integrate common features but can also be partly country-specific. For instance, bank lending rates must be obtained by spread equations over long-term nominal rates and may integrate country-specific channels to account for financial frictions. In contrast, models used for public finances and inflation forecasting are totally specific to the Banque de France.

Fourth, we use labor force projections from the French national institute of statistics and economic studies (Insee) and the long run equilibrium unemployment rate from an external assessment described in section 4.3.3, which deals with the definition of long-run output and of its components.

HICP inflation forecasts are performed within MAPI (Model for Analysis and Projection of Inflation in France, see de Charsonville et al., 2017, for a detailed presentation). MAPI conditional forecasts use FR-BDF's macroeconomic projections as inputs. FR-BDF conditional forecasts then use updated inflation forecasts from MAPI to determine the consumption price deflator, which is in practice exogenized. Both models iterate several times within an exercise to achieve convergence.

In the same spirit, public finances are projected outside FR-BDF, within a model known as MAPU and developed by the Banque de France's Public Finances Studies Division. MAPU conditional forecasts use FR-BDF projections as inputs and iterate with FR-BDF until convergence, in particular regarding the level of spending, revenues and deficit. Here, the difficulty lies in the discrepancy between public finance variables which are set on an annual basis, and quarterly national accounts which are the basis of FR-BDF. Public finance variables need to be adjusted at quarterly frequency, sometimes using external information

[72] In unconditional simulations, competitors' prices are partly determined by effective exchange rates and by *exogenous* competitors' prices in *foreign* currency. Crude oil prices' in euro depend on the dollar/euro nominal exchange rate.

about the timing of the implementation of fiscal measures. Finally, the minimum wage equation (51) is modified in conditional forecasting exercises in order to comply with the effective minimum wage formula, in particular regarding the long run stability condition that we introduced to preserve the balanced-growth path of the model. In conditional forecasts, the minimum wage equation is replaced by the following equation:

$$w_t^m = \delta_{q1}\Bigg(\pi_{4,C,t} + 0.5\left[\pi_{4,W,t} - \pi_{4,C,t}\right]\Bigg) + \epsilon_t \tag{131}$$

which closely follows the minimum wage formula. Finally, additional judgments are necessary in forecasting exercises, due to discrepancies between variables used in the model (consumer price inflation, average wage per capita) and those used by the true formula (HCPI for the bottom 20% of households, blue-collar real wage per capita).

Table 4.11.1: Summary of model changes for conditional projections

Variables	**Status in FR-BDF**	**Determination in conditional forecasts**
Euro-area output gap	Endogenous	Eurosystem assumptions
Euro-area GDP deflator	Endogenous	Eurosystem assumptions
World demand to French exporters (volume)	Exogenous	Eurosystem assumptions
Brent price in dollars	Exogenous	Eurosystem assumptions
Brent price in euros	Endogenous	Eurosystem assumptions
Exchange rates (dollar/euro, NEER)	Endogenous	Eurosystem assumptions
Competitors' prices for exports and imports in euros	Endogenous	Eurosystem assumptions
Short-term interest rate (3-month Euribor)	Exogenous	Eurosystem assumptions
Expected short-term interest rate (5-year futures of 3-month Euribor)	Exogenous	Eurosystem assumptions
Long-term interest rate (OAT 10 years)	Endogenous	Eurosystem assumptions
Long-term bank lending rate to firms	Endogenous	Auxiliary BLR model with Eurosystem assumptions
Long-term bank lending rate to households	Endogenous	Auxiliary BLR model with Eurosystem assumptions
HICP: Total, Core, Energy and Food	Endogenous	Auxiliary model for inflation forecasts of the Banque de France (MAPI)
Public finance variables: VA subcomponents, taxes and subventions, social contributions and transfers at current prices	Endogenous	Auxiliary model for Public finances forecasts of the Banque de France (MAPU)
Labor force	Exogenous	Insee projections
Long run equilibrium unemployment rate	Exogenous	Banque de France projections

Note : the word "exogenous" is used here for variables that do not depend on other endogenous variables of the model.

5 Main model properties under VAR-based expectations

In this section, we present the main model properties of the baseline version of the model, where expectations are assumed to be VAR-based.[73] In the first subsection, we describe the long run equilibrium and convergence mechanisms of FR-BDF. In the second subsection, we present impulse response functions (IRFs) for four demand shocks: a shock on the short run interest rate, a term-premium shock on the long run interest rate, a foreign demand shock, a government spending shock, and three supply shocks: an oil price shock, a cost-push shock and a permanent labor efficiency (or labor-augmenting productivity) shock.

5.1 Long run convergence in unconditional simulations

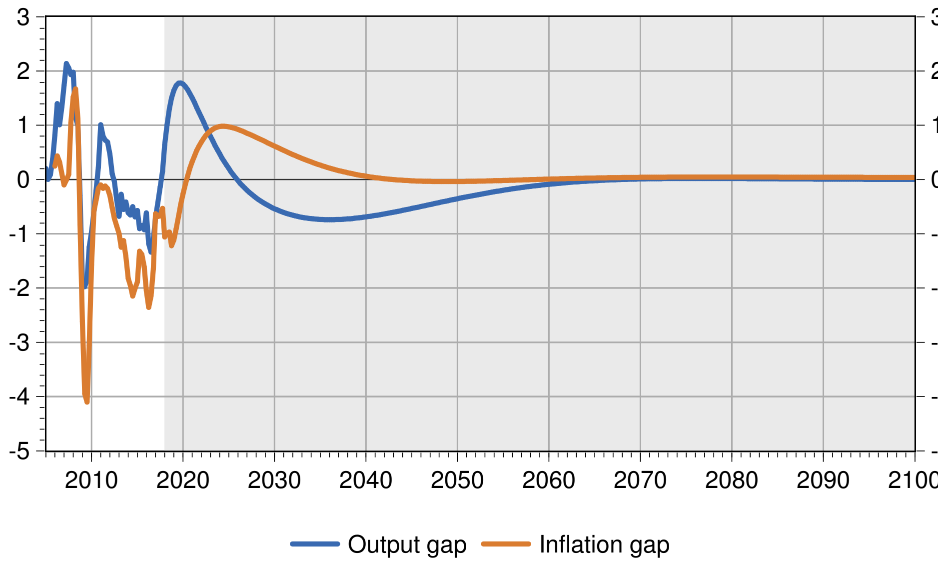
In this subsection, we start by checking the convergence of unconditional simulations of FR-BDF toward a balanced growth path (BGP). To do so, we run an unconditional simulation of the model under VAR-based expectations from 2018Q1 to 2300Q1. All residuals are set at 0 at the beginning of the simulation. Exogenous variables are extrapolated with the three key growth rates that prevail along the balanced growth path: $\Delta\bar{y}$ for real variables homogenous to output, $\Delta\bar{e}$ for real variables homogenous to labor productivity and $\bar{\pi}$ for nominal variables homogenous to a price. Based on these simple assumptions on exogenous variables, the point of this simulation is not to provide a realistic forecast along the transition, but to assess how fast the model converges towards the balanced growth path.

As shown in Figure 5.1.1, the output and inflation gaps – the inflation gap being defined as the gap between value added price inflation of market branches and its steady state level $\bar{\pi} = 1.9\%$ – converge toward 0 in around 40 years. In the short run, FR-BDF's dynamics can deviate from long run targets due to nominal and real rigidities modeled with PAC. As a result, convergence toward the BGP is achieved once both nominal and real rigidities have vanished. In particular, as explained in subsection 4.3 about the supply block, convergence of the output gap toward 0 requires convergence of prices and employment toward their targets. To understand such a slow convergence, we should keep in mind two key features of the model, on which we will come back below: in the absence of independent monetary policy, the closure of these gaps is only ensured by price-competitiveness mechanisms which affect other model dynamics very slowly; the dynamics of these variables are also influenced by stock variables, i.e. capital services and net financial assets, which have very inertial dynamics.

More precisely, the path of the output gap goes through the following phases. At first, it increases with a peak at almost 2% in 2020 mainly because of imports, which were initially above their target and then return to their equilibrium level. On the nominal side, in addition to the upward pressure created by this increase in demand, as interest rates – initially at low levels close to zero – converge toward values consistent with the steady state of the short-run rate which is calibrated at its pre-crisis average (annualized rate at 3.7%), the price target increases significantly with the cost of capital and the implied inflation results in a real appreciation (see Figure 5.1.2). This loss of price competitiveness pushes after 2020 the output gap downward to such an extent that an undershoot occurs before its long run

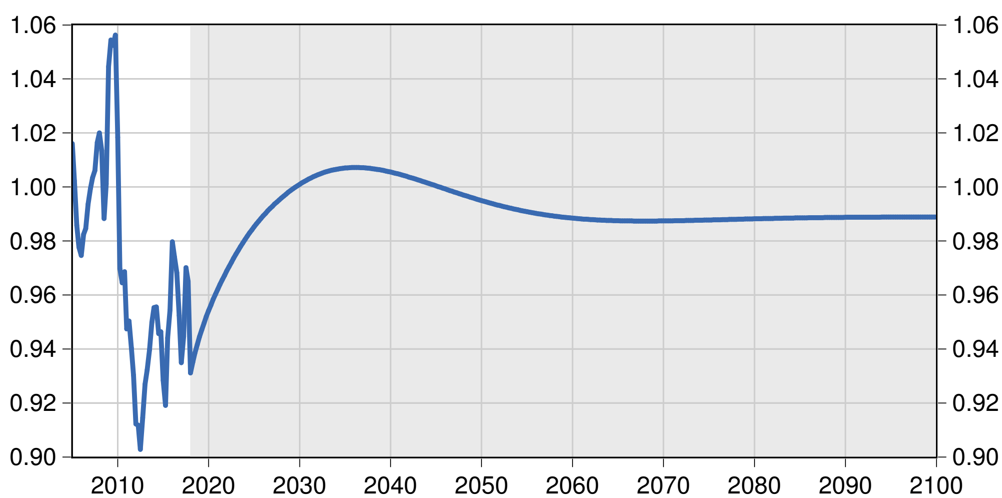
[73] This version of the model is close to the one used for conditional forecasting, except for some differences detailed in section 4.11.

Figure 5.1.1: Output and inflation gaps



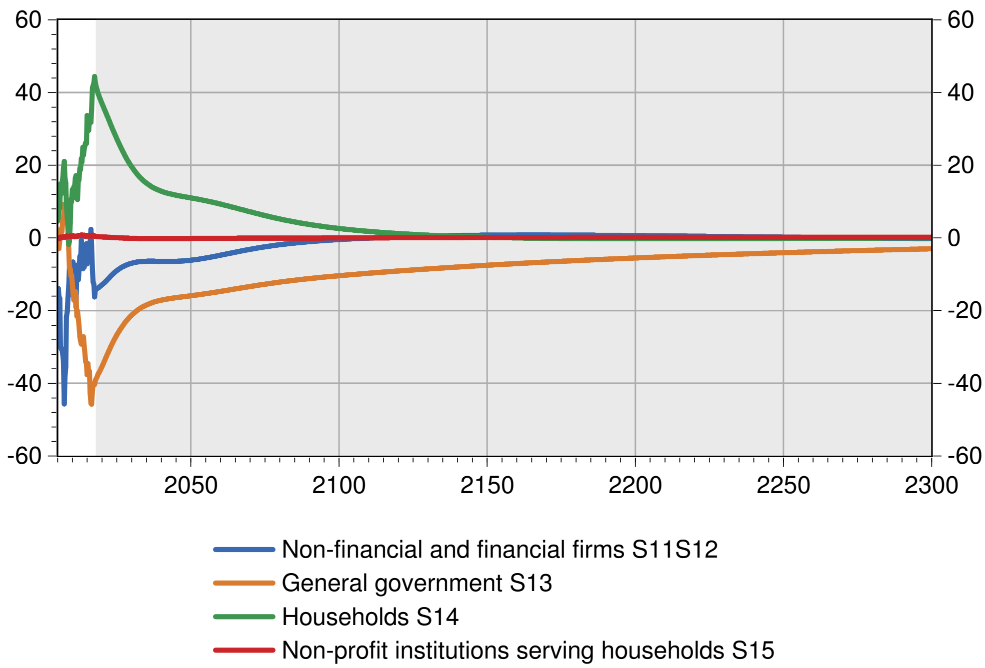
closure. Movements in the price level and inflation also amplify the effects of real interest rates on consumption and investment, because nominal interest rates are fixed, and slow down convergence toward the BGP. As shown by Nakamura & Steinsson (2014), this channel is crucial in explaining the effects of demand shocks in monetary unions, in particular after government spending shocks.

Figure 5.1.2: Real effective exchange rate, $P_{Q,t}/P_{cx,t}$



Finally, the convergence of net financial assets is illustrated in Figure 5.1.3. First, thanks to stabilization rules described in section 4.8.5 about net asset positions, net asset ratios of firms, non-profit institutions and government converge toward their steady state levels (respectively -70%, 2% and -40%, in percent of annualized GDP), although convergence is very slow in the case of government net assets. It should be kept in mind that these stabilization rules are based on the gaps between agents' net lending ratios and the corresponding debt-stabilizing ratios. With such a specification and the chosen calibration, these rules are non-aggressive, with the advantage of avoiding strong changes in the instruments in the short run and the drawback of such a slow convergence toward the long run.[74] Second, as shown in the same figure, households' net asset ratio (and consequently of net foreign debt)converges toward its steady state (around 120% in this simulation): this convergence is endogenously achieved in FR-BDF and critically depends on the marginal propensity to consume c.[75] In a simplified model of a small open economy, we can show that, in order to ensure that households do not accumulate wealth in a divergent way, households' marginal propensity to consume c should be larger than the threshold $(\bar{i} - g)/\bar{i}$, which depends on the steady state of the nominal interest rate $\bar{i}$ and on the long run growth rate of nominal output g.[76]

Figure 5.1.3: Net financial assets, deviation from the steady-state level in pp of annualized GDP



[74]The empirical analysis of the types of instrument used in the past by the French government for debt stabilization and the aggressiveness of such policies is left for further research.

[75]Because of hysteresis implied by the usage of chained prices, this long-run ratio of net assets of households might be sensitive to initial conditions of this simulation.

[76]A note with the detailed derivation of this condition in a simplified model is available upon request.

5.2 Impulse responses

We turn to short run dynamic properties of FR-BDF and present four demand shocks and three supply shocks. The four demand shocks are a standard monetary policy shock, a term premium shock, a foreign demand shock and a public consumption shock. The first two shocks will be analyzed briefly here and in more detail with a comparison of different expectations in section 6 as they are key to monetary policy transmission. The three supply shocks are a cost-push shock on VA price, an oil price shock and a (permanent) labor efficiency shock. All simulations are performed around the balanced-growth path of the model. We run an unconditional simulation from 2018Q1 to 2300Q1 which is our baseline, then we re-run an alternative simulation from 2149Q4 to 2300Q1 with the shock hitting the economy in 2150Q1. IRFs are then calculated as percentages or absolute deviations, depending on the type of variable, between the alternative and baseline scenarios.[77]

5.2.1 Short-term interest rate shock

The monetary policy shock is designed as a one period +100bp shock to the annualized short-term interest rate which is endogenously passed on to the long-term rate through the term structure equation and to the nominal effective exchange rate through the UIP. The shock transmits to the model through $\bar{i}_t$, the short-rate long-term expectation in E-SAT with an estimated historical persistence $\lambda_i = 0.92$; see section 3.1.2.

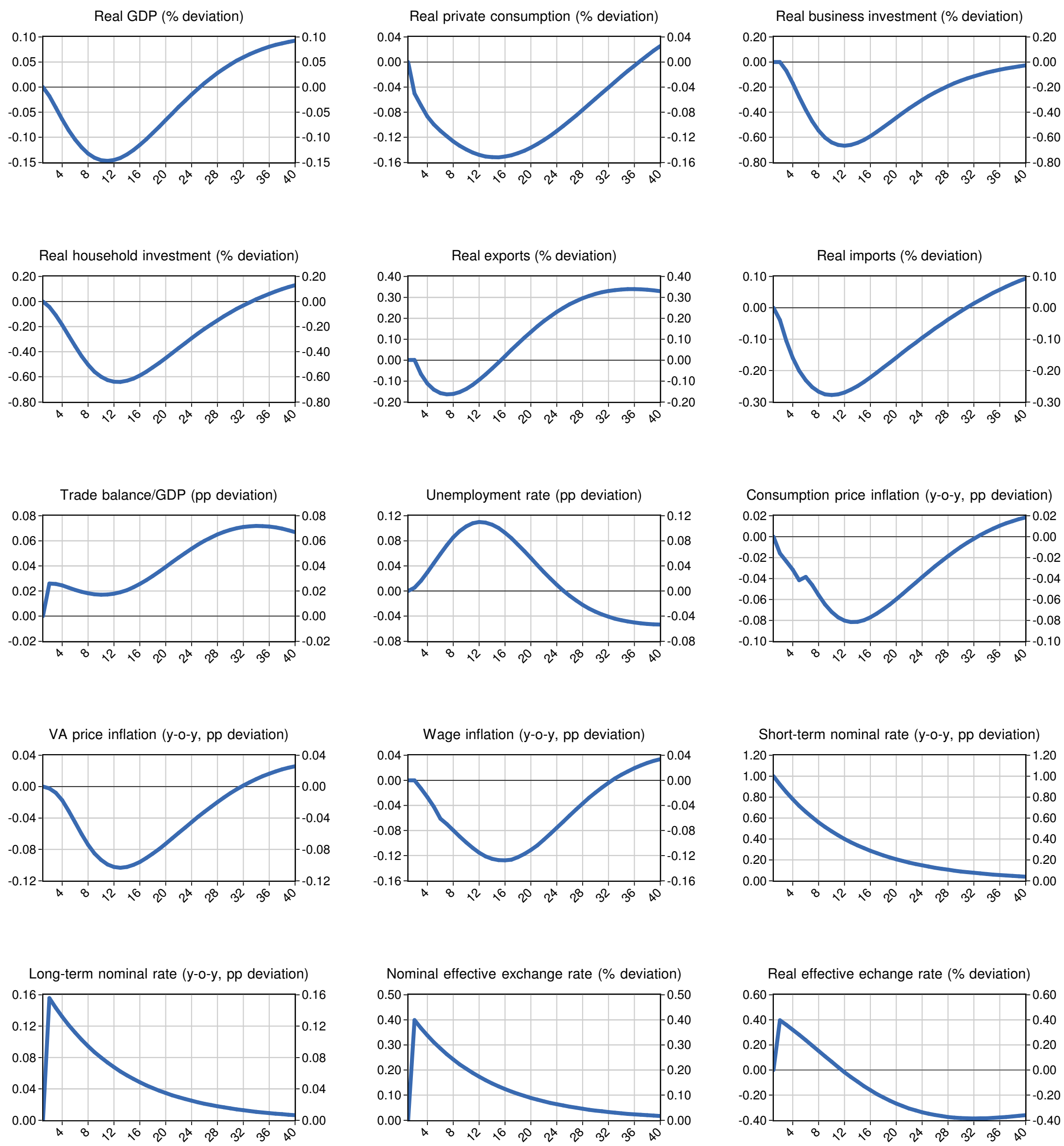
Figure 5.2.1 presents the short- to medium-term dynamics of core variables of FR-BDF in response to the short rate shock. First, as regards financial variables, we see that both the long rate and the nominal effective exchange rate[78] react one period after the shock, due to backward-looking expectations: the long rate increases by +0.16pp, while the nominal effective exchange rate appreciates by +0.40%. We observe a negative peak impact of -0.15% on real GDP, + 0.1pp on the unemployment rate and -0.10pp on the year-on-year VA price inflation after 12 quarters. This shock also transmit directly through expectations, since the short rate is a core variable in E-SAT. On the real side, the short rate strongly affects business and household investments through the real user cost of capital, through its effect on expected inflation and on long-rates. It also affects the target for household consumption both through the real interest rate and through households' expected income. Real exports are also hit in the short run by the exchange rate appreciation. On the nominal side, the short rate strongly affects nominal wage inflation and VA price respectively through the expected unemployment gap and the expected change in VA price target inflation. Disinflation rapidly improves price competitiveness and the real effective exchange rate depreciates with respect to the baseline scenario after 12 quarters. Net exports significantly increase, boosting real GDP above the baseline after 24 quarters.

FR-BDF shows a stronger response than our former model Mascotte, which can be related to the stronger sensitivity of investment to the cost of capital and to the widespread influence of the short-term rate through expectations, which play in particular a role here in term

[77] IRFs of inflation and interest rates as well as ratios with respect to GDP are calculated as absolute deviations.

[78] In our IRFs, we report the nominal effective exchange rate of French exporters using indirect quotation rather than direct quotation (i.e. an increase of the nominal exchange rate means that the domestic currency is appreciating).

Figure 5.2.1: Short-term interest rate shock

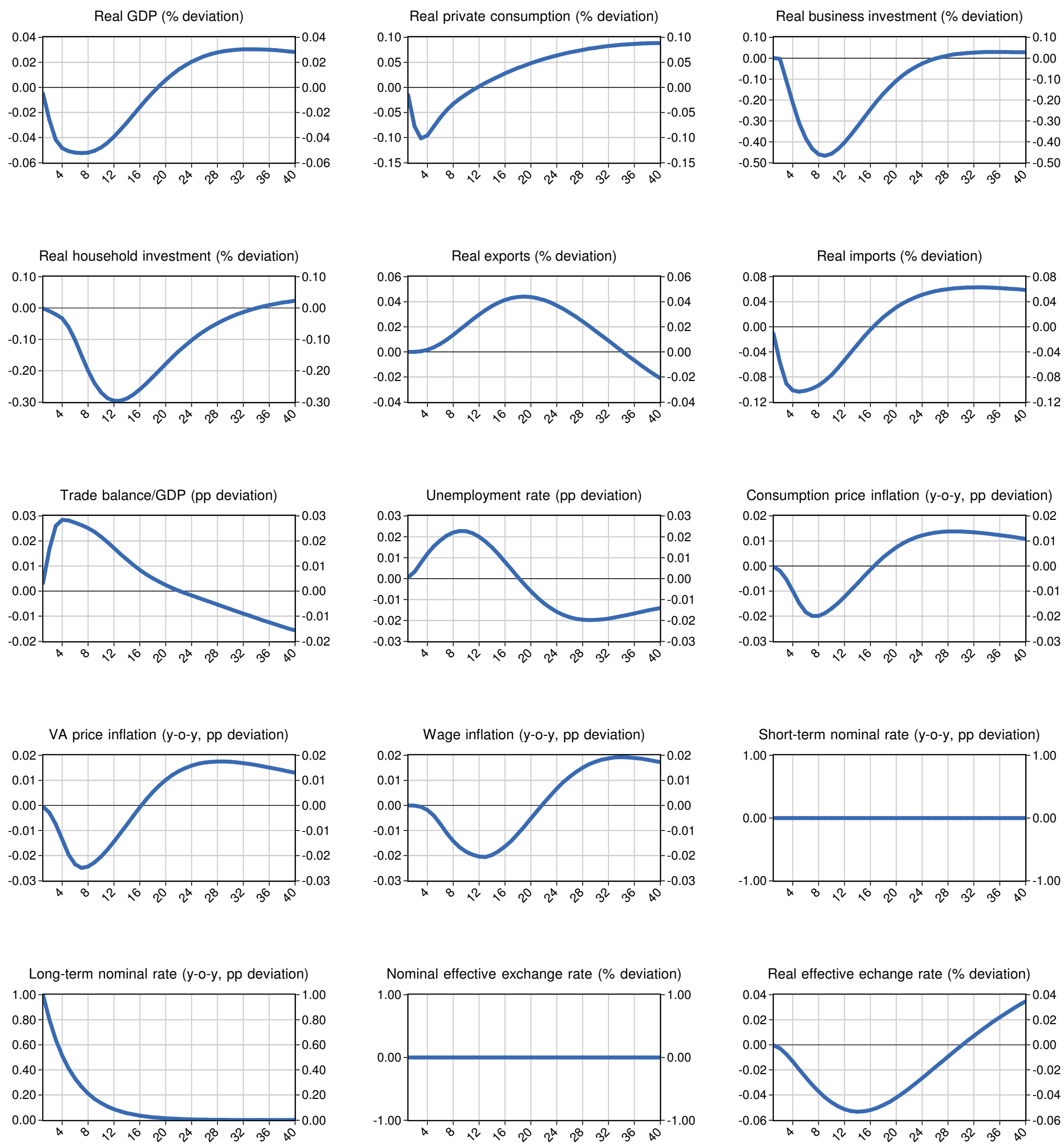


structure and uncovered interest rate parity equations. Even with these features, the response is weaker than in FRB/US: in addition to the absence of a reaction of the rest of the euro area, which would have amplified the French response through foreign demand had it been endogenous, this lower sensitivity could also be related to the absence of a wealth effect and a financial accelerator in France, two particularly channels important for the United States.[79]

[79] Regarding the financial accelerator, this channel appears in FRB/US through an effect of the expected output gap on risk premia. As we do not find a significant effect of this channel on French data, we do not include this channel in FR-BDF. Still, this result does not preclude that such a channel could play a role in France in stressed times. This topic is left for further research.

5.2.2 Term premium shock

Figure 5.2.2: Term-premium shock



The term premium shock is designed as a +100bp shock to the annualized nominal long-rate in the absence of any shock to the short-rate, with estimated historical persistence; see Table 4.8.2 in section 4.8. Contrary to with the short rate shock, the long rate has no direct effect on expectations; in particular, the nominal effective exchange rate is unaffected by the shock. The shock is endogenously passed on to bank-lending rates and to user costs of capital.

As shown in Figure 5.2.2, a +100bp term premium shock has a peak impact of -0.05%

on real GDP, +0.02pp on the unemployment rate and -0.025pp on VA price inflation. On the real side, the increase in the real user cost of capital for businesses and households causes a strong decrease in both types of investment, by -0.45% and -0.3% respectively. On the nominal side, Phillips effects cause a slowdown in VA price, wage and consumer price inflation. The negative effect on inflation creates a real depreciation which in turn boosts net exports in the short run. Contrary to the short rate shock, the term-premium shock has a modest negative impact on real private consumption in the short run but a positive impact in the medium to long run, due to the increase in real income.

5.2.3 Foreign demand shock

The foreign demand shock is designed as a one-period +1% shock on the volume of foreign demand addressed to French exporters, with a persistence calibrated at $\rho = 0.9$. We use this persistence for shocks to exogenous variables.

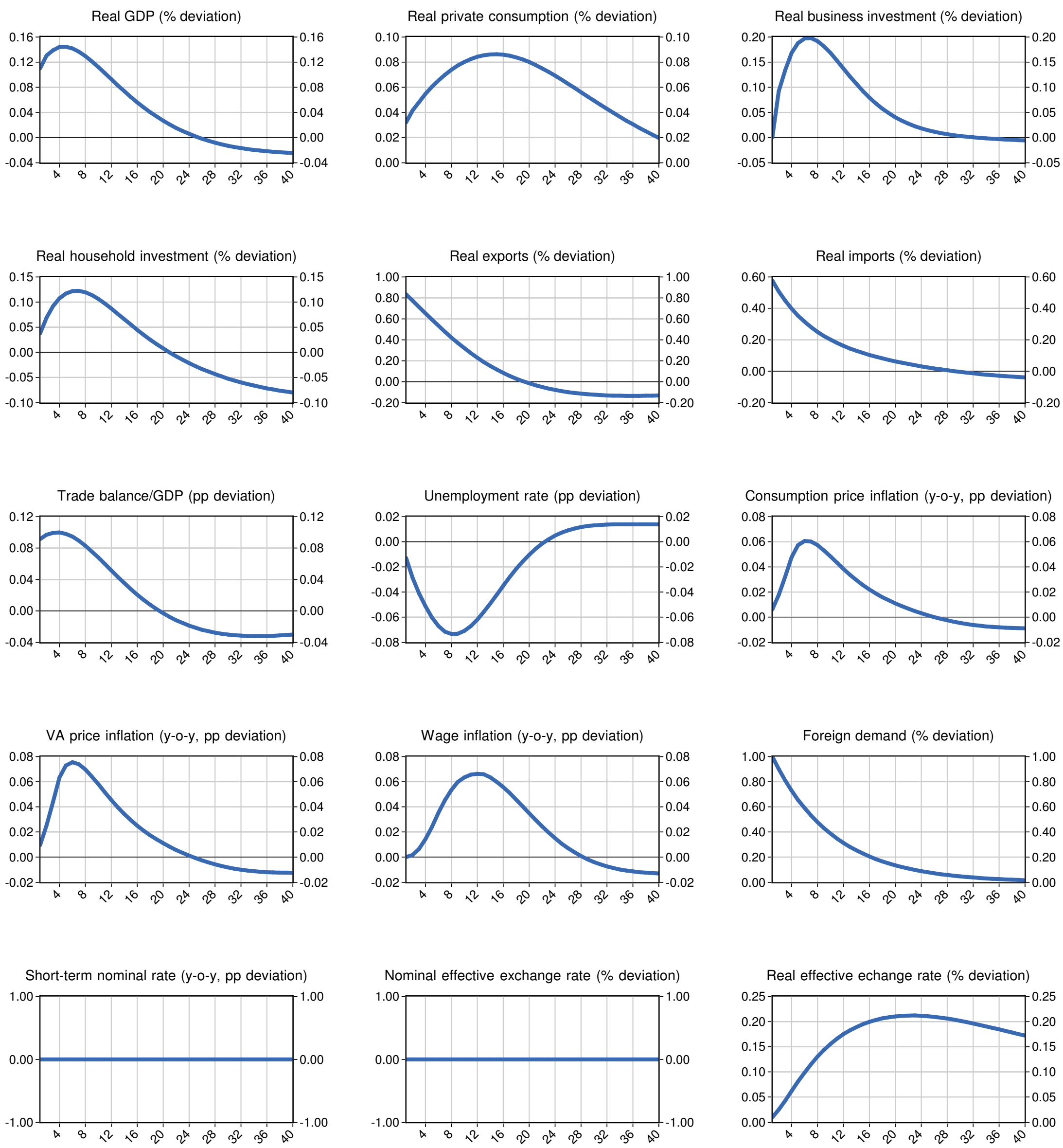
Results are presented in Figure 5.2.3. After 4 quarters, the foreign demand shock has a peak impact of +0.14% on real GDP, -0.07pp on the unemployment rate and almost +0.075pp on VA price inflation. On the real side, the main short run driver of real GDP growth is net exports, with the trade balance improving by +0.1pp of nominal GDP. Real exports and imports increase respectively by +0.8% and +0.6% on impact. This strong response of imports is related to two factors: (i) the large import content of exports (around 33%) taken into account in the total import adjusted demand (IAD); (ii) the large short-run elasticity of imports excluding energy to IAD (1.9). Households' private consumption and investment also increases on impact, by 0.03% and 0.04% respectively, due to the GDP growth gap in their respective short run equations which account for rule-of-thumb agents. Finally, the shock has a peak impact of +0.2% on real business investment. On the nominal side, we observe positive effects on both VA price and consumer price inflation respectively, due to Phillips effects in the short run VA price equation. After 10 quarters, wage inflation increases by up to +0.06pp due to the decrease in the unemployment rate. Finally, the shock effect on real GDP turns negative after 24 quarters, due to the appreciation of the real effective exchange rate.

5.2.4 Government spending shock

The government spending shock is designed as an *ex post* +1% of GDP shock to impact on public consumption, with a persistence calibrated at $\rho = 0.9$.

As shown in Figure 5.2.4, real GDP increases by around +1.2% on impact and then progressively reverts to zero and turns negative after 18 quarters. Both unemployment and VA price inflation have hump-shaped responses to the shock. Unemployment decreases by -0.1pp on impact, with a peak impact of -0.45pp six quarters after the shock. VA price inflation increases by 0.1pp on impact and reaches +0.5pp after five quarters. On the real side, household consumption and investment increases on impact, by 0.3% and 0.4% respectively. Net exports decrease strongly on impact, through the direct effect on real imports but also through the real exchange rate appreciation, due to higher VA price inflation. On the nominal side, the strong impact on VA price inflation and consumer price inflation reduces the real user cost of capital for both households and businesses. More importantly, wages react

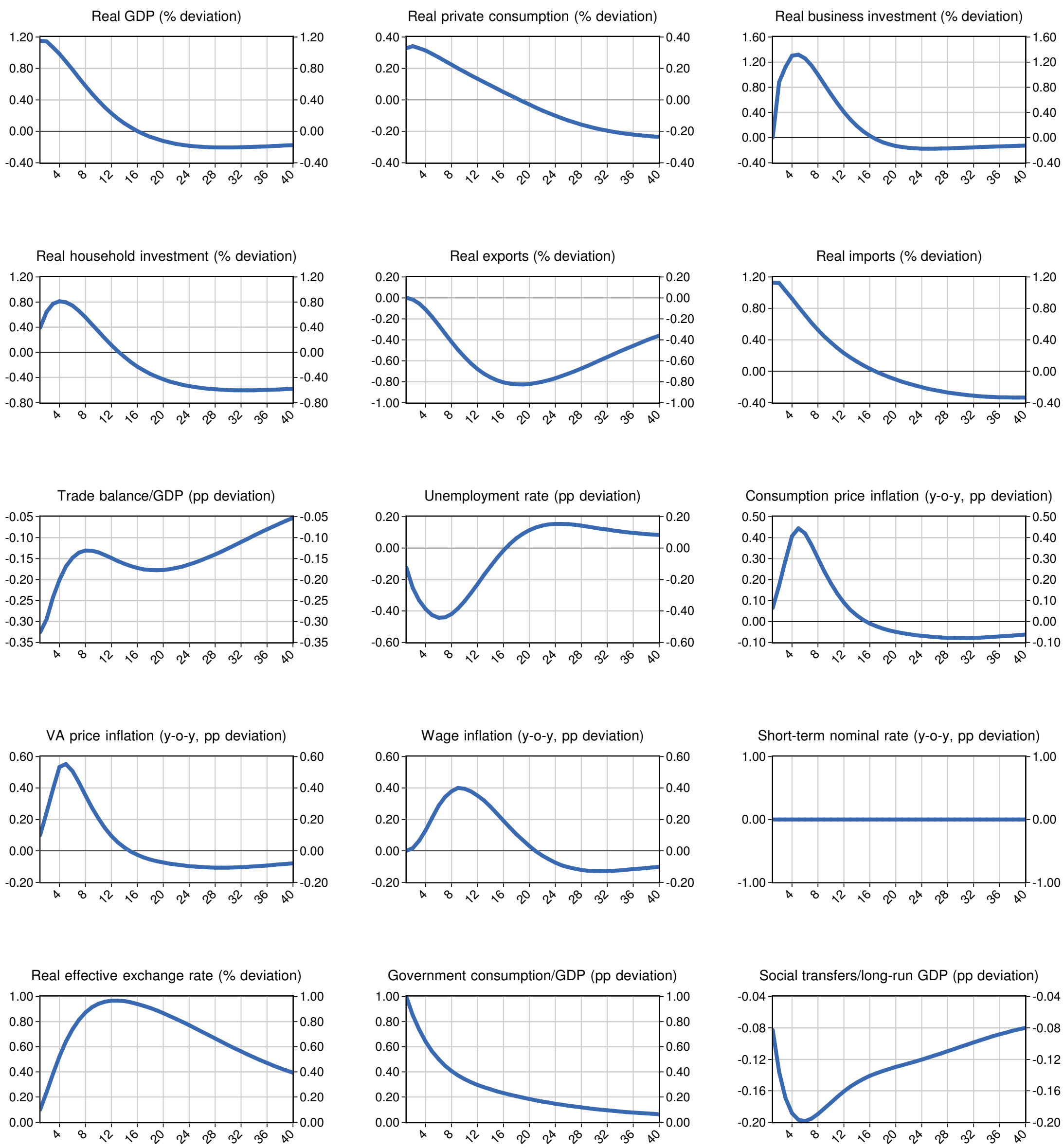
Figure 5.2.3: Foreign demand shock



less than both VA price and consumer price inflation, which reduces the real labour cost and boosts employment through demand in the short run, but also reduces real household income and consumption 16 quarters after the shock.

The size of the government spending multiplier, above 1 at impact in FR-BDF, is broadly in line with those found in the empirical literature. Notable features are a positive crowding-in effect on both consumption and investment in the short run. Empirical literature usually finds higher multipliers in monetary unions or fixed-exchange rate economies, because the nominal interest rate does not react (much) to regional or national inflation (Nakamura & Steinsson, 2014). More generally, fiscal multipliers can be much higher when monetary policy

Figure 5.2.4: Government spending shock

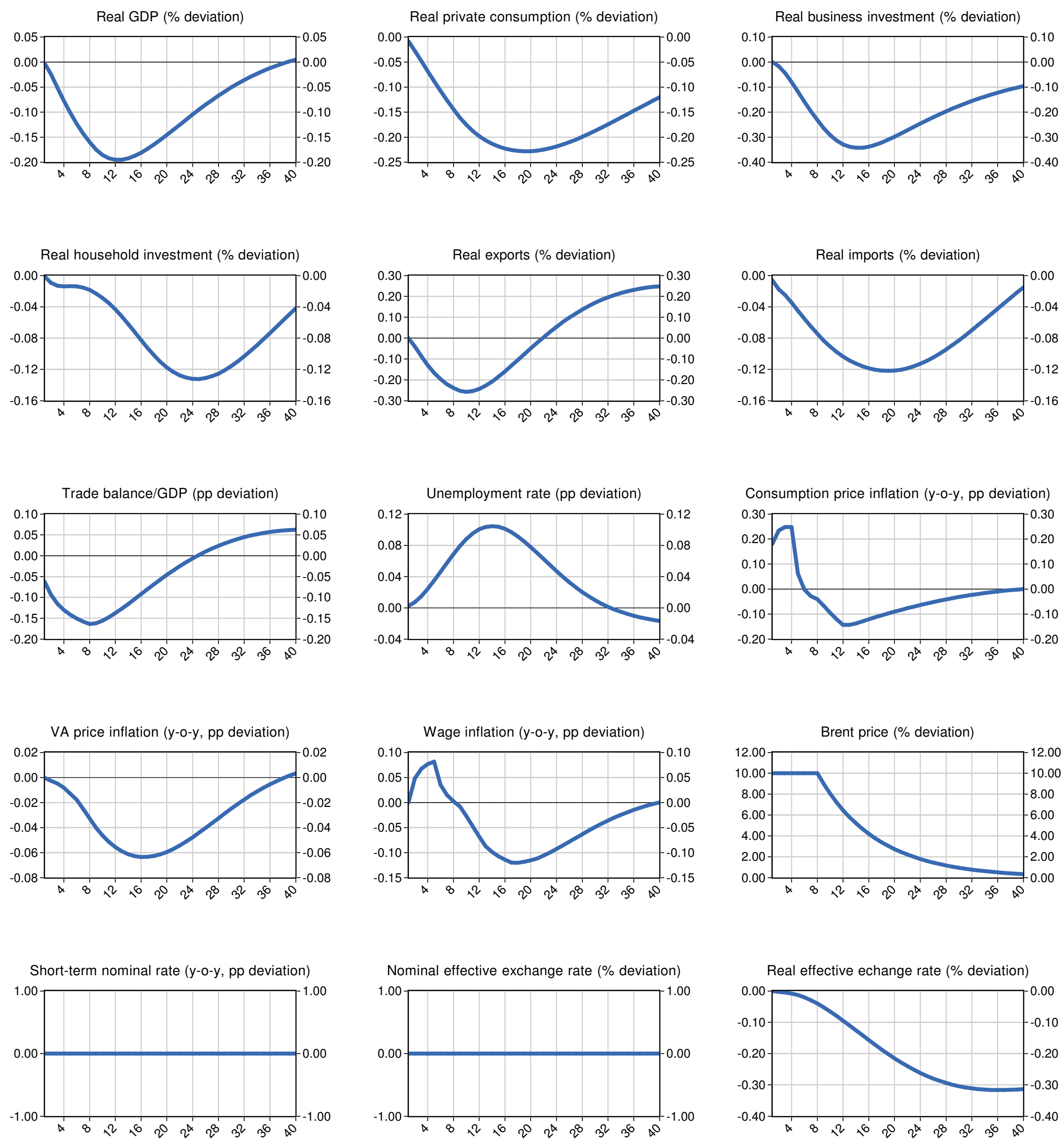


is "passive" or at the zero lower bound (Leeper et al., 2017).

In contrast, when Farhi & Werning (2016) assume that households are Ricardian and government spending is financed by a budget-neutral rule on lump-sum transfers, they find some crowding out in an open economy DSGE model in monetary union, because of the real appreciation plus expected reversal of the price level. However, in an other case, they assume the presence of non-Ricardian households and outside-financed government spending, which leads to crowding in. This case seems more similar to our setup, as we have: (i) non-Ricardian effects in the short-run equation of consumption thanks to the inclusion of the current change of the output gap; (ii) non-aggressive fiscal rule on social transfers.

5.2.5 Oil price shock

Figure 5.2.5: Oil price shock



The oil price shock is designed as a +10% shock to the Brent oil price (in 2014 euros, the shock is equivalent to +4.6 euros), during 8 quarters, with a persistence calibrated at $\rho = 0.9$ afterward. More importantly, the shock affects the economy through the energy import price. There is no direct effect on the VA price through the Factor Price Frontier (FPF). Finally, foreign competitors' prices are left unchanged, i.e. the shock is asymmetric.

Results are shown in Figure 5.2.5. After 12 quarters, the shock has a peak impact of -0.2% on real GDP, +0.1pp on unemployment and -0.06pp on VA price inflation. On the nominal side, the main effect occurs through the energy import price and consumer inflation, with a peak impact of +0.25pp on the latter. At first, wages react incompletely and with a lag to the increase in consumer price inflation, which will reduce real household income in the short run. Wage inflation then declines by -0.12pp after 16 quarters, due to the rise in unemployment. In contrast, the response of VA price inflation shows an absence of second-round effects: VA price inflation does not react on impact but will decrease thereafter by 0.06pp, due to Phillips effects from the real side. On the one hand, households' purchasing power will decline in the short run, due to the strong increase in the consumer price, while on the other, the real labor cost will rise and reduce labor-demand. On the real side, all aggregate demand components fall after the shock. Household consumption and investment decrease progressively due to lower purchasing power and higher unemployment. Real business investment decreases due to the decline in the VA of market branches. Finally, net exports decline in the short run (-0.25% after 10 quarters) mostly due to the import content of exports.[80] Meanwhile, real imports only fall progressively to -0.12%, along with aggregate demand components. In the medium run, the decrease in the VA price progressively restores price competitiveness and real exports rise above the baseline after 22 quarters.

5.2.6 Cost-push shock

Our cost-push shock is designed as a markup shock such that the VA price increases by 1% on impact and is then endogenously passed on to the economy according to the persistence of the VA price short run equation. As the VA price is a core variable in E-SAT, the shock will affect PAC equations through expectation terms.[81]

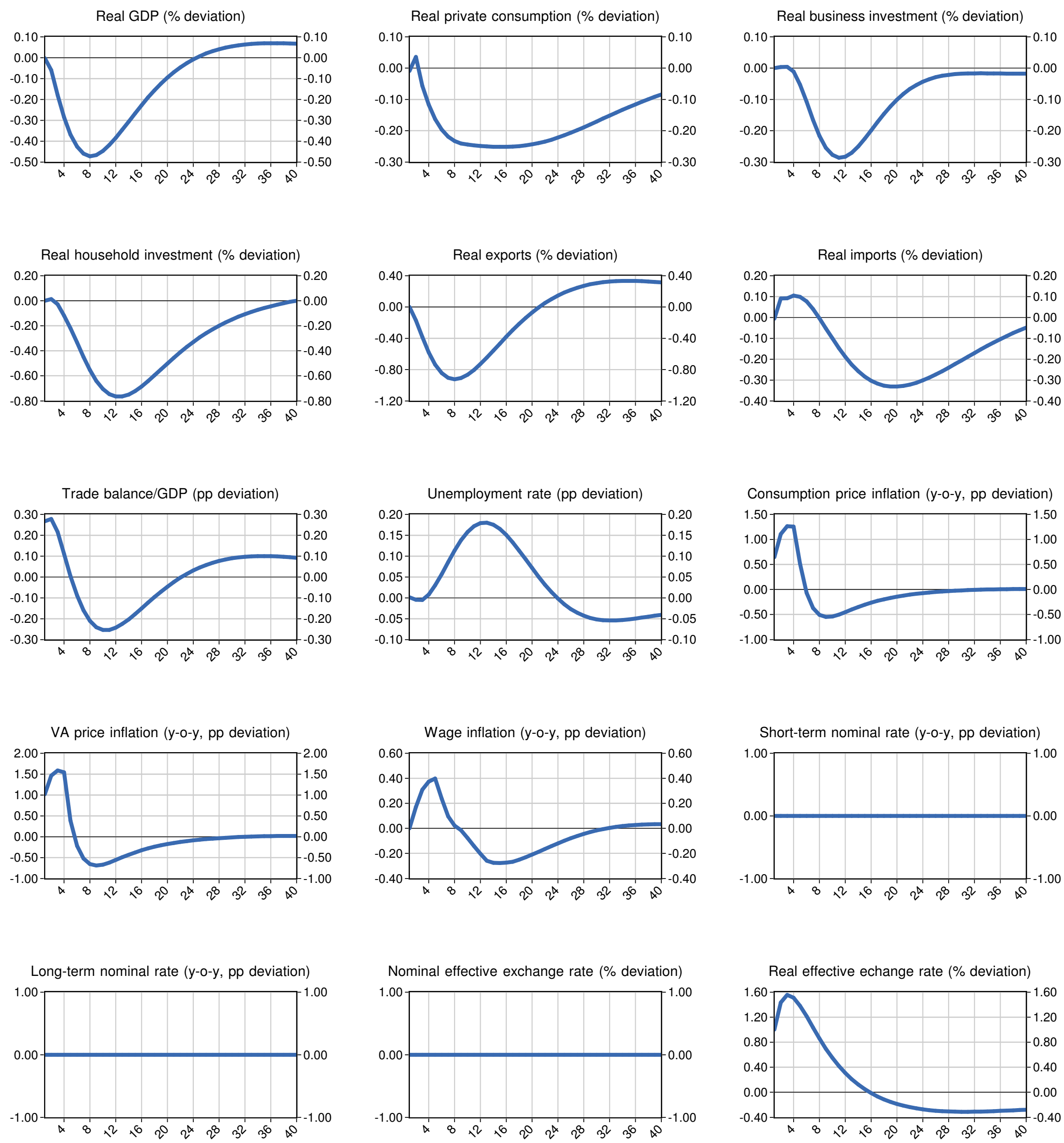
Results are shown in Figure 5.2.6. On impact, the shock materializes by a +1pp increase in VA price inflation. After a peak impact of +1.5pp after four quarters, VA price inflation rapidly decreases and stays below the baseline during around 20 quarters. It has a peak impact of -0.45% on real GDP after eight quarters and +0.2pp on unemployment after 12 quarters. On the nominal side, consumer price inflation follows a similar path as VA price inflation: it increases by +0.6pp on impact and rapidly decreases to 0 after 64 quarters. Lower inflation with respect to the baseline after four quarters is necessary for the price level to adjust downward and for the output gap to close through external trade (see below). As after the oil price shock, wage inflation underreacts to consumer price inflation due to incomplete indexation in the wage Phillips-curve. On the real side, household consumption and investment are strongly and negatively affected by the decrease in expected real income. Both sub-components of real investment display a similar response. At first, their responses are dampened by the fall in the real user cost of capital, implied by higher inflation. The negative effect on market branches' VA and on households real expected income then reduces both types of investment demand.[82] As regards the external trade sector, the trade balance

[80]The export price strongly depends on the import price both in the long run and in the short run, due to the import content of exports. As a result, a shock to the energy import price increases the export price and reduces the price competitiveness of exports.

[81]The shock is technically engineered as a one-period 1% shock on the residual of VA price short run equation.

[82]The stronger effect on household investment is due to the higher persistence in the household investment equation compared to business investment, which amplifies the initial shock.

Figure 5.2.6: Cost-push shock



temporarily increases after the shock, as a result of the real exchange rate appreciation effect on imports but then decreases after four quarters. Finally, the real effective exchange rate depreciation progressively restores price competitiveness and net exports, which boosts real GDP.

5.2.7 Labor efficiency shock

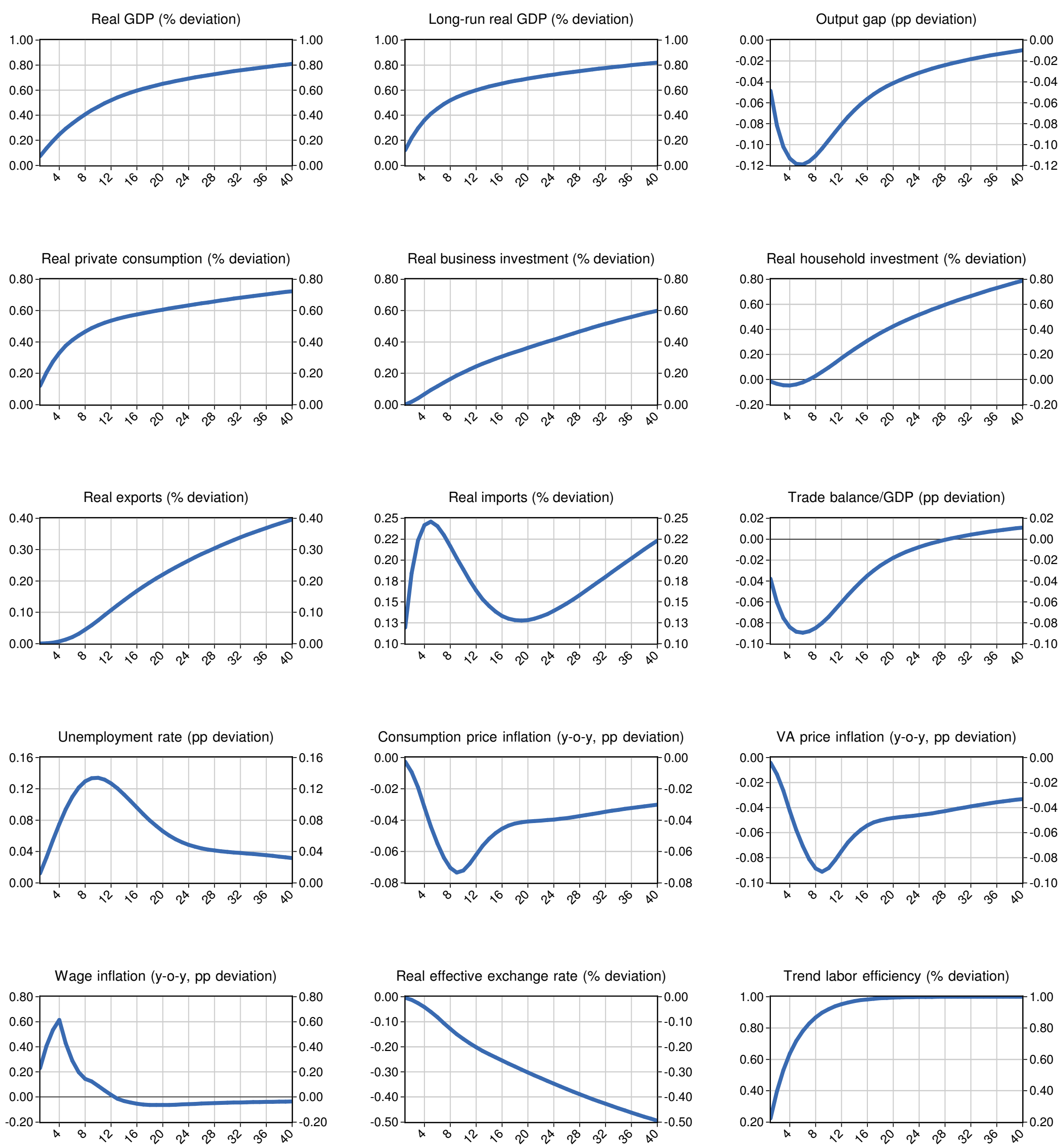
The labor efficiency shock is designed as a *permanent* +1% shock to the level of trend labor efficiency, with estimated historical persistence.[83] The shock's main transmission channels are long run output, VA price and labor demand targets. Through its impact on long run ouput and on the output gap, it also affects all expectations terms. Given that it is a permanent shock to the *level* of trend labor efficiency, it naturally has permanent effects with respect to the baseline scenario. Still, the growth rate of long run output is unchanged at the BGP. As a result, IRFs describe the transitory dynamics toward the new BGP.

Figure 5.2.7 shows the results. Initially, the shock has a positive and progressive impact on real GDP and long run real GDP, with a larger effect on the latter resulting in a negative output gap. The productivity shock reduces labor demand, leading to an increase in unemployment. It then progressively declines with the rise in output. On the real side, all components of domestic demand progressively increase. Long run real GDP directly drives expected real income and both household consumption and investment. Business investment reacts with lags to the shock, mostly due to a higher real user cost of capital, but progressively rises with market branches' real value-added. The trade balance first deteriorates on account of the increase in imports, in line with domestic demand. But real exports then progressively increase thanks to the real exchange rate depreciation. On the nominal side, the productivity shock reduces VA price inflation by -0.09pp and consumer price inflation by -0.07pp after eight quarters. In the meantime, wage inflation rises by about +0.2pp on impact (up to +0.6pp four quarters after the shock) because wages increase one-to-one with trend labor efficiency in our Wage-PC (see equation 52).

Finally, the real exchange rate continuously depreciates: following the shock, the domestic price level is permanently reduced. In particular, VA price inflation stays persistently below the baseline up to 40 quarters, due to high inertia of employment. As a result, the positive unemployment gap slows wage inflation persistently by around -0.05pp 20 quarters after the shock, which explains why inflation remains below the baseline although the output gap has closed.

[83] The shock is implemented as a permanent shock to the residual of the trend labor efficiency equation, described in section 4.3.

Figure 5.2.7: Labor efficiency shock



6 Monetary policy transmission under different expectation assumptions

6.1 Alternative version of FR-BDF with model-consistent expectations

We apply FR-BDF with model-consistent expectations for two purposes: (i) to better understand the VAR-based case and the behavior of the model; and (ii) to study policy issues where forward-looking behavior matters, as in e.g. a forward guidance exercise.[84]

6.1.1 Equations of present value variables under model-consistent expectations

First, we present the equations for the present values with a constant discounting scheme.[85] They can be written in the forward-looking way with a single lead, see subsection 3.3.2. The discounting weights sum to unity in all cases, except for the present value of the expected non-discounted sums of future short run interest rates and future foreign short run interest rates.

$$PV(i)_t = (1 - 0.97)i_t + 0.97PV(i)_{t+1} \tag{132}$$

$$PV_{nd}(i - \bar{i})_t = i_t - \bar{i} + PV_{nd}(i - \bar{i})_{t+1} \tag{133}$$

$$PV_{nd}(i_F - \bar{i})_t = i_{F,t} - \bar{i} + PV_{nd}(i_F - \bar{i})_{t+1} \tag{134}$$

$$PV(\pi_Q)_t = (1 - 0.994)\pi_{Q,t} + 0.994PV(\pi_Q)_{t+1} \tag{135}$$

$$PV(y_H)_t = (1 - 0.95)y_{H,t} + \frac{0.95}{exp(\Delta\bar{y})}PV(y_H)_{t+1} \tag{136}$$

$$PV(\hat{u})_t = (1 - 0.98)\hat{u}_t + 0.98PV(\hat{u})_{t+1} \tag{137}$$

Second, we present equations for the present values of expected changes in targets which appear in PAC equations. Their parameters are determined by the structure and parameters of the corresponding short run equation and discount factor, see subsection 3.3.1.[86] The

[84]For forecasting only VAR expectations are applied as it is easier to apply judgment.

[85]The choice of discount factors is explained in the sections of the corresponding variables, i.e. in the subsection 4.8 about the financial block for expected interest rates, in the subsection 4.6.2 about business investment for expected inflation, in the subsection 4.6.1 about consumption for expected income and in the subsection 4.5.1 about labor supply for expected unemployment.

[86]The discount factor of PAC equations is generally calibrated at 0.98, as explained in the subsection 4.1 about the estimation approach.

expectation is defined as a weighted sum of a variable over future quarters.

$$\begin{aligned}
PV(\pi^*_Q)_t = \;& \beta_0 \quad \pi^*_{Q,t} + \beta_1\pi^*_{Q,t+1} + \beta_2 PV(\pi^*_Q)_{t+1} + \beta_3 PV(\pi^*_Q)_{t+2} && (138)\\
PV(\Delta n^*_S)_t = \;& \beta_0 \quad \Delta n^*_{S,t} + \beta_1\Delta n^*_{S,t+1} + \beta_2\Delta n^*_{S,t+2} + \beta_3\Delta n^*_{S,t+3} + \beta_4 PV(\Delta n^*_S)_{t+1} && \\
& + \quad \beta_5 PV(\Delta n^*_S)_{t+2} + \beta_6 PV(\Delta n^*_S)_{t+3} + \beta_7 PV(\Delta n^*_S)_{t+4} && (139)\\
PV(\Delta c^*)_t = \;& \beta_0 \quad \Delta c^*_t + \beta_1\Delta c^*_{t+1} + \beta_2 PV(\Delta c^*)_{t+1} + \beta_3 PV(\Delta c^*)_{t+2} && (140)\\
PV(\Delta logI^*_H)_t = \;& \beta_0 \quad \Delta logI^*_{H,t} + \beta_1\Delta logI^*_{H,t+1} + \beta_2 PV(\Delta logI^*_H)_{t+1} && \\
& + \quad \beta_3 PV(\Delta logI^*_H)_{t+2} && (141)\\
PV(\Delta logI^*_B)_t = \;& \beta_0 \quad \Delta logI^*_{B,t} + \beta_1\Delta logI^*_{B,t+1} + \beta_2\Delta logI^*_{B,t+2} && (142)\\
& + \quad \beta_3 PV(\Delta logI^*_B)_{t+1} + \beta_4 PV(\Delta logI^*_B)_{t+2} + \beta_5 PV(\Delta logI^*_B)_{t+3} &&
\end{aligned}$$

The estimated parameters are shown in Table 6.1.1.

Table 6.1.1: Coefficients of equations of present values in the model-consistent case

	$PV(\pi^*_Q)_t$ Coef.	$PV(\Delta n^*_S)_t$ Coef.	$PV(\Delta c^*)_t$ Coef.	$PV(\Delta logI^*_H)_t$ Coef.	$PV(\Delta logI^*_B)_t$ Coef.
β_0	0.06	0.06	0.12	0.06	0.08
β_1	-0.03	-0.05	0.01	-0.03	-0.02
β_2	1.41	0.02	0.75	1.53	-0.02
β_3	-0.48	-0.01	0.07	-0.59	1.18
β_4	-	1.77	-	-	-0.09
β_5	-	-1.11	-	-	-0.18
β_6	-	0.45	-	-	-
β_7	-	-0.17	-	-	-

Implementation In order to simulate the FR-BDF model under MCE, we first compile the VAR-based version, then remove the objects that stand for expectations and replace them with the equations above. For example, in the case of the consumption targer, in order to move from VAR-based case to MCE case, we remove the following expression

$$\mathrm{PV}^2\left(y_H - \bar{y}\right)_{t|t-1} + \alpha_1\left(\mathrm{PV}\left(i_H\right)_{t|t-1} - \left(\mathrm{PV}\left(\bar{i}\right)_{t|t-1} - \mathrm{PV}\left(\bar{\pi}\right)_{t|t-1}\right)\right)$$

from the short run equation of household consumption (62) and replace it with equation (140).

6.1.2 Simulation methodology

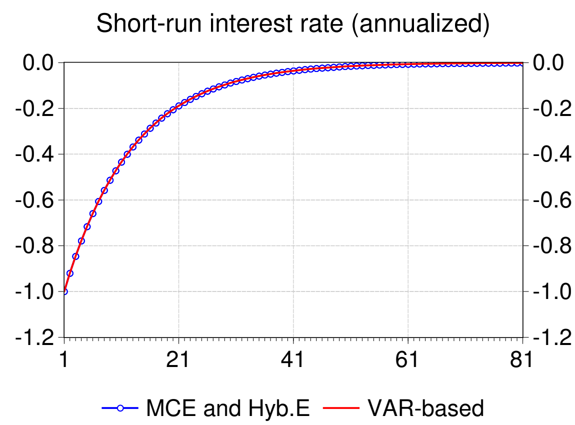
Counterfactual experiments under model-consistent expectations are computed in deviation from a baseline because FR-BDF is a non-linear model.[87] In order to be able to compare cases

[87] An example of non-linearity is the application of the log operator to the cost of capital.

under different expectation assumptions the baselines should be identical. This condition defines that the procedure we apply for unconditional simulations in the MCE case: it is done by inverting the MCE model and solving for "new" expectations and residuals on the historical sample, leaving observed variables unchanged. It should also be mentioned here that the MCE version is solved under perfect foresight and we do not take into account any uncertainty.

6.2 Monetary policy shock

Figure 6.2.1: Annualized short run interest rate after the monetary policy shock



The exercise In this section, we consider the response of FR-BDF to a 100bp shock to the annualized short run interest rate. The shock occurs in the future, sufficiently far ahead in order to consider that the model is at the steady state. The shock is temporary and lasts one quarter, see Figure 6.2.1. The short run interest rate follows an AR(1) process with a persistence parameter of 0.92, the one of the Taylor rule estimated with the expectation satellite model E-SAT. The euro area variables are exogenous.[88] In this section all the responses are presented in deviation from a baseline in pp.

This exercise is simulated under three expectation assumptions: (i) agents construct their expectations using the E-SAT model (VAR-based case), (ii) agents are forward-looking (MCE case) and (iii) Hybrid case (Hyb.E), where financial agents are forward-looking, i.e., their expectations are model-consistent, and the others use limited information to predict the future, i.e., their expectations are VAR-based. To be more specific, financial variables that are forward-looking in the Hybrid case include (i) the expected sum of discounted short run interest rates (which influences the long run interest rate), (ii) the expected sum of non-discounted short run interest rates (which influences the exchange rate), (iii) the expected sum of non-discounted foreign short run interest rates (which influences the exchange rate).[89] Table 6.2.1 summarizes the differences between the expectation assumptions.

[88] This is a strong yet important assumption, as euro area variables enter E-SAT and are hence included in the policy function of expected variables in the VAR-based case.

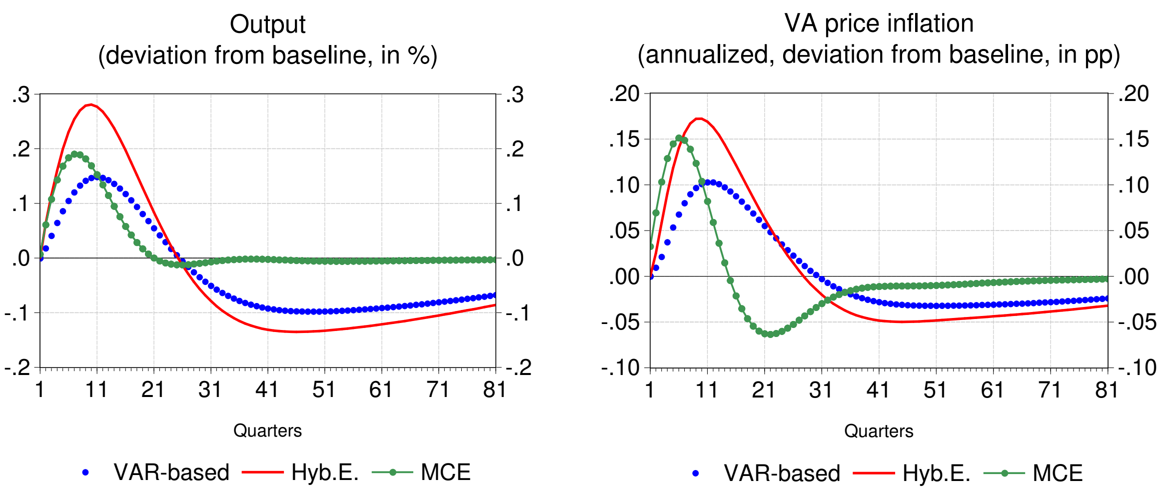
[89] However, the latter is exogenous and does not matter for the exercise.

Table 6.2.1: Differences between three expectation assumptions

	Financial variables	Non-financial variables
Hybrid	forward-looking	backward-looking
MCE	forward-looking	forward-looking
VAR-based	backward-looking	backward-looking

The propagation mechanism is the same under all expectation assumptions. First, a decline in the short run interest rate results in a decrease in the long run interest rate, which is the main driver of the bank lending rate and cost of capital, encouraging agents to invest. As regards external trade, exports rise due to the depreciation in the domestic currency caused by the difference between domestic and foreign short run interest rates. This, in turn, leads to a boom in the labor market and an increase in real disposable income – one of the main drivers of household consumption. The price response is positive and very persistent, which leads to a loss in competitiveness, decreasing exports in the medium run and downward pressure on the economy.

Figure 6.2.2: Monetary policy responses under different types of expectations

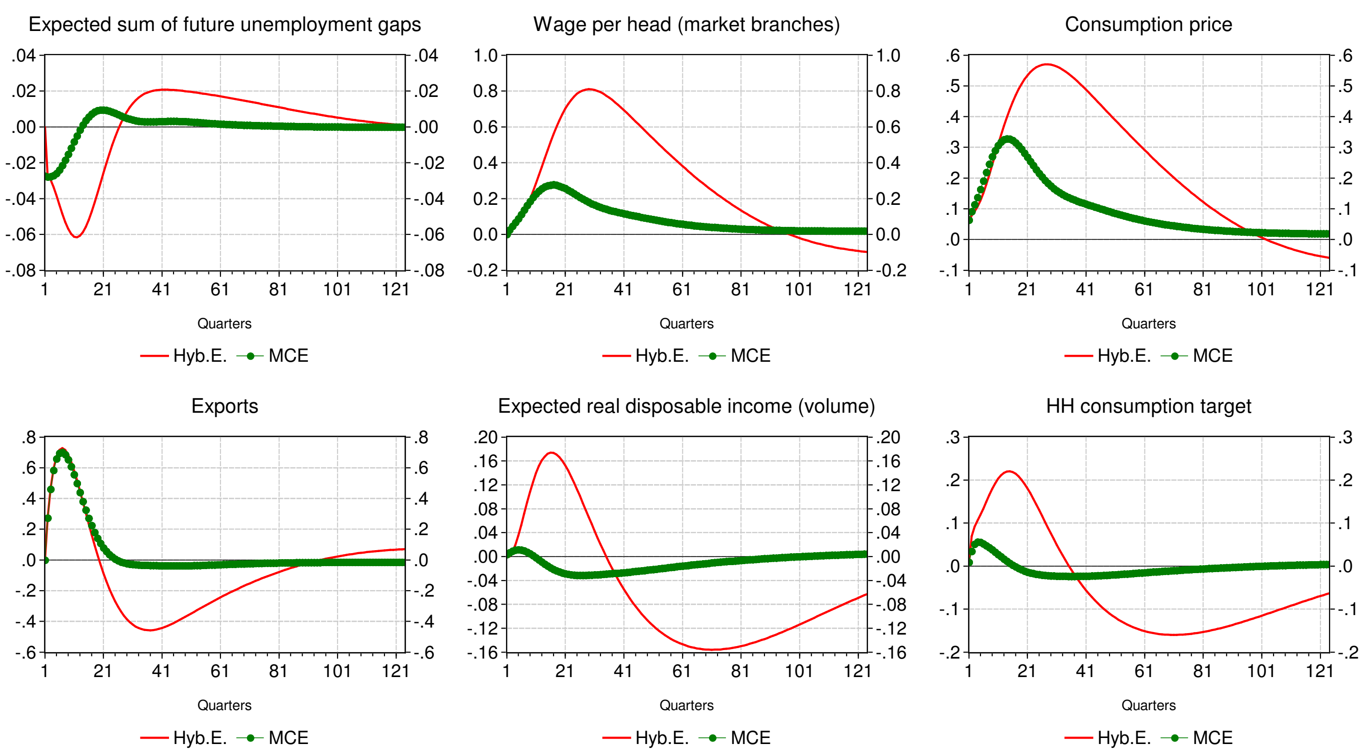


Main results The output and the value-added inflation responses are shown in Figure 6.2.2, the GDP components are available in Figure A.2 in the Appendix. All three cases are different in amplitude and speed of convergence. Three conclusions can be reached at this stage. First, we observe that the GDP impact response in the full MCE case is below that in the Hybrid case. This leads us to conclude that when non-financial variables are modeled in a forward-looking manner, they create a dampening effect in comparison to a VAR-based case. Second, the GDP response in the Hybrid case is larger than in the VAR-based case. This leads us to believe that forward-looking financial variables create an amplification effect in comparison to the VAR-based case. Third, convergence is much slower in the cases where non-financial variables are backward-looking with a long-lasting "undershoot" of demand components (VAR-based and Hybrid cases). The rest of this section discusses these points in detail.

This last difference in responses to an interest rate shock (long-lasting undershoot under VAR-based expectations but not under MCE) is similar to a difference pointed out in Roeger & Herz (2012) in the case of a closed-economy model with a Phillips curve that is either forward-looking or backward-looking and accelerationist.[90]

Dampening effect of forward-looking non-financial variables. The reason for this dampening effect is that backward-looking agents are more optimistic. They form their expectations using E-SAT, which predicts higher economic growth after the same shock. In E-SAT, the output gap reaches 0.34% at the peak (see Figure 3.1.2), while in the MCE case, it stands at 0.17% at the peak. In order to understand the implication of this "optimism", we compare responses under the Hybrid and MCE cases (see Table 6.2.1 for a reminder on the differences between them).

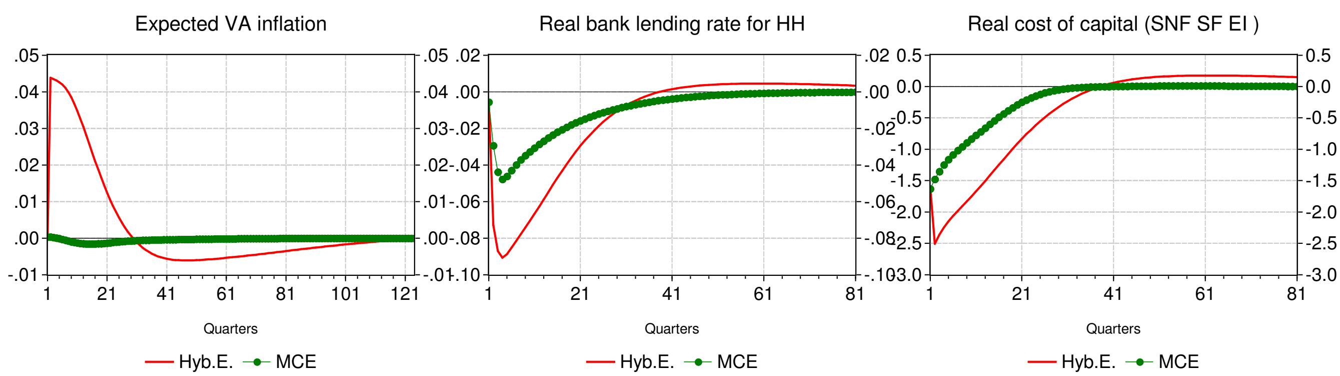
Figure 6.2.3: Monetary policy responses – 1st focus on Hyb.E. versus MCE



[90]In this paper, authors note that the forward-looking case would be more realistic for the United States, given that they do net find an overshoot in impulse responses of a SVAR estimated on US data. Our setup is quite different: we have an anchored Phillips curve and an open economy setup in a monetary union. It is unclear for us at this stage which response would be empirically more appropriate on French data.

The higher growth expected by the VAR-based non-financial agents (Hybrid case) leads to (i) lower expectations regarding the unemployment gap and (ii) higher expectations regarding the value-added inflation in comparison to the MCE case. Acting through the wage Phillips curve, lower expectations regarding the unemployment gap imply higher wages per capita and – via the price/wage loop – higher price level, see Figure 6.2.3.[91] In the short run, this leads to a larger increase in expected disposable income in the Hybrid case in comparison to the MCE case, which explains the differences in household consumption targets since the latter is one of their main components, see Figure A.2.[92] In the medium run since the price level stays positive, there is a loss in competitiveness and, hence, a decrease in exports. In the Hybrid case, the loss is so significant that it produces undershooting in exports and, through the labor market, in all the demand components. In the MCE case forward-looking agents are aware of the future loss in income flows. They start smoothing their consumption from the beginning (see the comparison of the expected disposable income in Figure 6.2.3).

Figure 6.2.4: Monetary policy responses – 2nd focus on Hyb.E. versus MCE



The larger positive response of expected value added inflation in the Hybrid case explains the difference in business and household investment when non-financial agents have different expectation assumptions, see Figure 6.2.4. Higher expected value-added inflation pushes the real bank lending rate and the real cost of capital down, while their nominal counterparts display the same response in both cases since the long run interest rate response is the same due to the fact that the financial variables share the same expectation formation.

Amplification effect of forward-looking financial variables. Model-consistent expectations of the short run interest rate are more reactive than backward-looking expectations

[91]Similar results were found in Mankiw & Reis (2002). Backward-looking price setting delivers a more inertial pattern of inflation compared to forward-looking behavior. Mankiw & Reis (2002), in particular, emphasize this difference between the New Keynesian Phillips Curve (which associates sticky prices and rational expectations) and the "backward-looking" Phillips Curve (associating sticky prices and adaptive expectations.

[92]Note that in the short run employment behaves similarly in both cases.

mainly because of the assumption that euro area variables are exogenous.[93] Indeed, under VAR-based expectations, even if EA variables are exogenized, the sensitivity of expectation components to the long run interest rate and exchange rates to the euro area short run rate comes from our VAR model (E-SAT), where the euro area is endogenous. In this VAR, a cut of the short run interest rate generates a boom of the output gap and inflation and, hence, a monetary policy tightening in the medium run. As agents expect this tightening, expectation components, which are loosely speaking averages of future short run rates, decrease less than if the short run rate would have reverted with historical persistence without any endogenous response of monetary policy to macroeconomic conditions (which is what happens under model-consistent expectations).

The difference in the short run interest rate expectations leads to differences in expectations regarding (i) the long run interest rate and (ii) the exchange rate, see Figure 6.2.5. Since the long run interest rate is the main driver of the nominal bank lending rate and the WACC, investment is more attractive in the case of forward-looking expectations of financial variables (Hybrid case).[94] Stronger depreciation in the Hybrid case boosts exports and an increase in the latter also contributes to the boom in the labor market and income flows. Note here that the difference in real exchange rates is mainly explained by the difference in the nominal rate. However, due to a higher cost of capital in the Hybrid case, the price increase is also stronger, which further amplifies the difference in real exchange rates.

6.3 Forward guidance[95]

6.3.1 Theoretical background for the forward guidance puzzle

Baseline DSGE models generically predict a very strong reaction of output and inflation to forward guidance policies. In addition, they predict that the further away the date when the interest rate is promised to decrease, the larger the immediate increase in GDP and inflation. If anything, one would expect promises of future interest rate cuts to be less powerful than current interest-rate cuts. This peculiar prediction of DSGE models has come to be known as the forward guidance puzzle, as coined by Del Negro et al. (2012). The puzzle states that the further away the date when the interest rate is promised to increase, the larger the immediate effect on household behavior and thus on inflation. In this section, we investigate this phenomenon using FR-BDF and a pair of DSGE models:

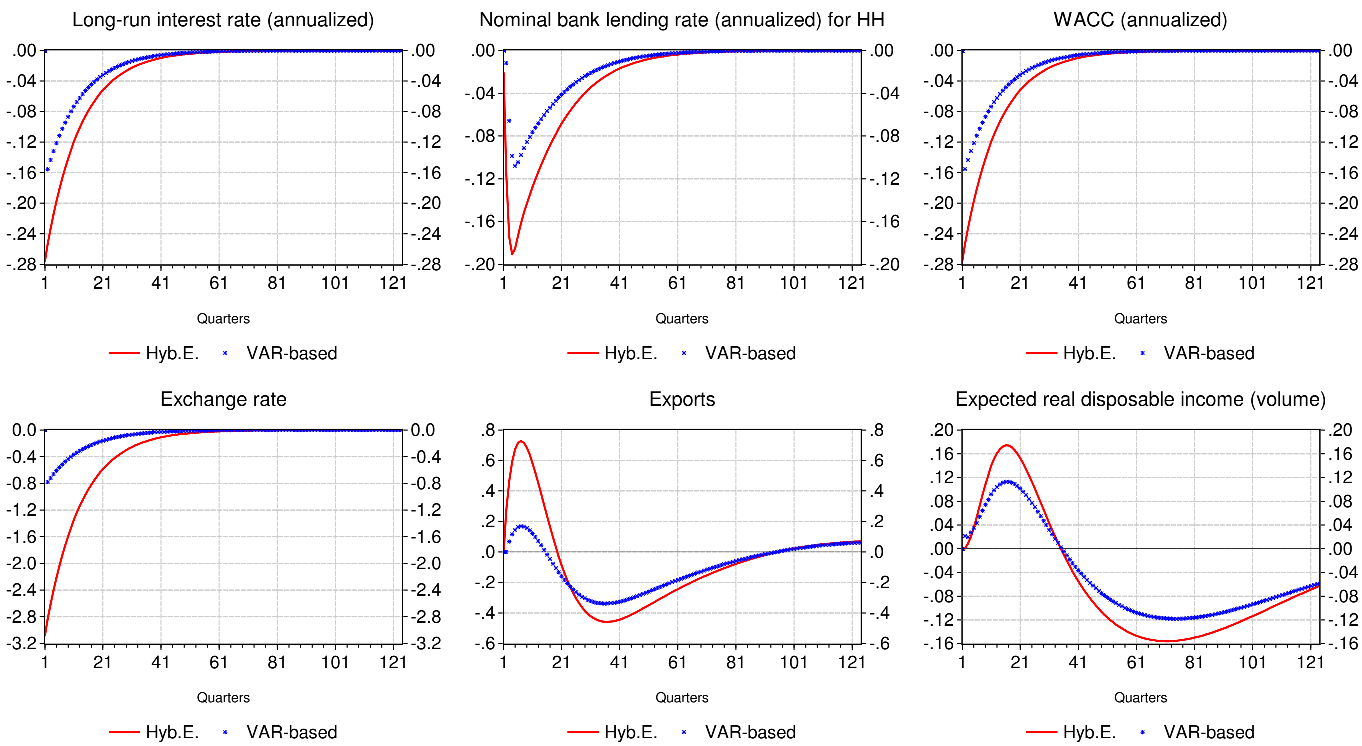
- NK 3-equation model both under a standard calibration (denoted "nondiscounted NK") and an alternative one (denoted "discounted NK") which assumes that house-

[93] Even though we have equations for the euro area variables, the foreign block does not react to them and we do not wish to simulate the model with endogenous euro area variables in a part of it (expectation variables) while keeping them exogenous in other parts (foreign demand and prices). In order to fully endogenize the role of the euro area, we would need to connect the model to a model of the euro area. In this case the model would retain the same expectations irrespective of whether financial variables are forward-looking or not. This exercise is left for further research. Meanwhile, we analyze the effect of the assumption of exogenous euro area variables on our results and, to do so, we compare the VAR-based case with the Hybrid case.

[94] Note that inflation expectations are more or less the same in both cases (they are backward-looking).

[95] We would like to thank Julien Matheron and Stephane Dupraz for providing the simulation results for the FPH model and for offering helpful comments.

Figure 6.2.5: Monetary policy responses – focus on Hyb.E. versus VAR-based



holds do not react as much to interest-rate cuts far in the future because they discount future consumption, which McKay et al. (2017) argue could arise e.g. if some households are credit constrained. This model is characterized by equations (143), (144) and an inertial Taylor rule.

- The DSGE model of Woodford & Xie (2019) where agents have finite planning horizons, denoted FPH. The average planning horizon is set to four quarters.

McKay et al. (2016) provide a clear explanation based on the intertemporal IS and Phillips curves. For the sake of simplicity we provide expressions for these curves below. A linearized form of the Euler equation, as presented in e.g. Galí (2008) is given by:

$$c_t = \mathrm{E}_t c_{t+1} - \sigma_1 \left(r_t - \bar{r} \right) \tag{143}$$

where r_t is the real short rate and c_t stands for consumption. Note that it is common to rewrite (143) with respect to the output gap x_t instead of c_t to obtain what is called the intertemporal IS curve. In a more complex model, such as the one in Smets & Wouters (2007), the equation would contain additional terms such as lagged consumption due to habit formation in utility and a term measuring the disutility of labor. For the simulation exercise we modify this equation in the alternative case ("discounted NK") by multiplying the first term on the right hand side by a discount factor $\nu = 0.97$.

The New Keynesian Phillips curve relates inflation to the output gap and expected inflation. The standard formulation is:

$$\pi_t = \beta_1 \mathrm{E}_t \pi_{t+1} + \kappa x_t \tag{144}$$

As with Euler equations, more complicated models have more complicated analogous expressions, e.g. in Smets & Wouters (2007) an additional lagged inflation term appears in the equation due to indexation of prices. To account for rigidities on the labor market, they also have a wage Phillips curve.

First, assuming that the real interest rate is constant, note that the Phillips curve (144) can be re-expressed as a discounted infinite sum of future output gaps and the IS curve (143) as the sum of future short rates:

$$\begin{aligned} \pi_t &= \kappa \sum_{j=0}^{\infty} \beta^j \mathrm{E}_t x_{t+j}, \\ x_t &= -\sigma_1 \sum_{j=0}^{\infty} \left(r_{t+j} - \bar{r}\right). \end{aligned}$$

Suppose the central bank engineers today an increase of 1pp in the short rate r_t, resulting in an immediate fall in x_t by σ_1 and a move in π_t by $\kappa\sigma_1$. On the other hand, if the interest rate increase occurs at some date in the future, the response is larger. For example, an increase promised in the infinite future would have an immediate contemporaneous effect on inflation of $\kappa\sigma_1/\left(1-\beta\right)$. If the discount factor $\beta = 0.99$, this response would be 100 times greater than the response to a contemporaneous increase.

Second, if the nominal rate instead of the real rate is kept constant before the shock, there is another amplification channel with inflation feeding back to the real rate. The inflation response at the date of the shock in the future is also negative, but the closer the current date, the larger the inflation response, as the output gap is expected to remain negative. Then consumption continues to decrease when solving backward – from the date of the shock to the current date – as the real interest rate rises, driven by falling inflation. This will amplify the negative response of inflation even further. In order to overcome this puzzle, a standard DSGE model has to be modified such that an additional discount parameter appears in front of the short run interest rate.[96]

As shown below with a simulation exercise, we do not observe such a puzzle in FR-BDF. The reason for this comes from the real side rather than the nominal one, since in FR-BDF prices and wages are modeled in a similar manner to the DSGE approach, i.e. assuming price and wage stickiness. While FR-BDF does not have an explicit Euler equation, it does have an equation relating current desired consumption to permanent income $y_{H,t}$ and the premium of the real household bank lending rate $r_{LH,t}$ over the trend of the real short rate $\bar{r}_t$:

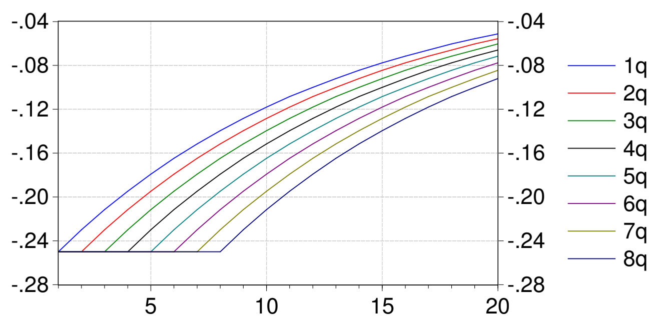
$$c_t = \alpha + y_{H,t} - \sigma_2 \left(r_{LH,t} - \bar{r}_t\right) \tag{145}$$

[96]Such modifications include e.g. heterogenous agents (McKay et al., 2016), a weakly credible central bank (Coenen & Wieland, 2004), overlapping generations (Del Negro et al., 2012) and finite planning horizon (Gabaix, 2016).

The key conceptual difference between the two demand curves – (143) and (145) – is that in FR-BDF consumption depends on the household lending premium instead of the short rate. The former is constructed using the 10-year government bond, which itself depends on a *discounted* term structure equation – i.e. an equation describing the 10-year rate with a term premium and a discounted sum of future short rates R_{t+j}. Because of this discounted term structure, the later the announced shock occurs, the smaller the effect of the future short run interest rate on the household bank lending rate. In addition, (145) departs from assumptions regarding the parametrization made in standard DSGEs by assuming that the discount factor used to construct the permanent income term $y_{H,t}$ (0.95 or equivalently an annual discount rate of 25%) is low compared to values consistent with observed interest rates[97] and that the risk aversion term implied by the term $\sigma_2 \approx 0.55$ is at a high value, around 10, compared to estimates of the DSGE literature, which generally lie in a range going from 1 to 5.

6.3.2 Quantitative comparison

Figure 6.3.1: Shock to the short run interest rate (annualized, in pp)



In order to illustrate the point, we present simulations of FR-BDF under model-consistent expectations and a fully exogenized euro area in which the annualized interest rate is lowered by 25bp with respect to its steady state for one to eight periods respectively, see Figure 6.3.1. The short run interest rate follows a simplified AR(1) rule to avoid complications related to interactions with the euro area. We also assume that there are no changes in the patterns of intra euro area trade, either in volumes or prices. Real GDP growth and consumer price inflation are computed as year on year changes.

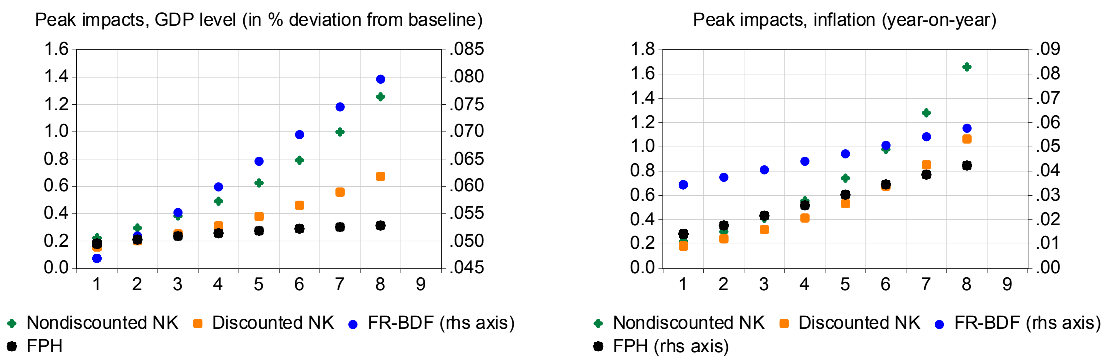
The results of the exercises are shown in Figure 6.3.2. Comparing all models, two results stand out. First, for a standard monetary policy shock – a shock that lasts only one quarter and contains no forward guidance – the peak output response is weaker in FR-BDF than in all the DSGE models considered (responses from FR-BDF are plotted on the right-hand axis). The same remark applies for inflation although the model with Finite Planning Horizons (FPH) delivers a peak impact that resembles that of FR-BDF.

[97] This is due to attempts to approximate terms relating to uncertainty and risk aversion. See Section 4.6.1 or Reifschneider (1996) for details.

Second, FR-BDF does not suffer from the forward guidance puzzle, and non-baseline DSGE models provide either a partial or full solution to the puzzle.[98] In FR-BDF, the peak impacts on GDP and inflation are almost perfectly linear in the duration of the forward-guidance announcement. Among DSGE models, all variants of the baseline DSGE model attenuate the response to forward-guidance announcements, although the additional discounting only partially solves the puzzle. In FPH, the impacts are even concave in the horizon of the announcement: the effect of forward guidance fades with the horizon.

These results are sensitive to specific details relating to the specification and parametrization of the models. In FR-BDF, agents are implicitly characterized by a very high degree of risk aversion. In most DSGEs this elasticity is calibrated to values close to 1. In unreported simulations, the peak effects of output and inflation in the FPH model are virtually identical to those of FR-BDF if it is calibrated with a risk aversion coefficient ten times as large as in the baseline calibration. In this case, the elasticity of output to the real interest rate has a similar magnitude in FPH as in FR-BDF.

Figure 6.3.2: Responses of GDP and Inflation to Forward Guidance Policies



Note: The figures plot the peak effect on GDP (in percentage points deviation from steady-state) and year-on-year inflation (in percentage points) of the announcement that interest rates will be 25bp lower for n = 1, ..., 8 quarters. Non-discounted NK is a simple DSGE model subject to the forward guidance puzzle. Discounted NK is the DSGE model that embeds the idea of credit-constrained households, as suggested in McKay et al. (2017). FPH is the DSGE model of Woodford & Xie (2019) with a finite planning horizon set on average to 4 quarters.

[98]Note that our characterization of the absence of the puzzle, i.e. a linearity of peak responses with respect to durations of shocks instead of an exponential curvature, is somewhat different from that found in the academic literature, where the puzzle is analyzed by studying the dependence of the magnitude of contemporaneous effects on the how far in the future is the date of the actual policy change, which is typically assumed to last only for a single period. In this context the puzzle concerns merely the fact that the magnitude is increasing. The structure of our experiment and the characterization was chosen due to its closer proximity to actual policy applications, where central banks promise to keep the interest rates constant for some specific period.

6.4 Asset purchase programmes[99]

With interest rates at the effective lower bound, the standard monetary policy tools have reached their limit within the euro area. In their place, the Eurosystem has applied several unconventional tools, including a variety of asset purchase programmes (APP). These programmes aim to provide monetary accommodation by transferring liquidity directly to market participants in exchange for various assets, including government bonds, thus bypassing the traditional lending and money creation channels. The empirical literature has identified effects on not only term premia, but also on exchange rates.

We use the estimated response of financial variables to study the effects of these policies on the French economy between 2015 and 2018 via simulations of FR-BDF. To do so, we construct a counterfactual outcome by removing the effects of APP on the term premium and exchange rates. The simulations are conducted both under VAR-based and model-consistent expectations. In the main experiments we assume that France is economically disconnected from the rest of the euro area, e.g. there are no changes in foreign prices or foreign demand from the rest of the euro area.

To obtain an assessment of the effects of APP on the French economy we use a sequence of shocks to the term premium and to the nominal exchange rate.[100] The shocks on term premium are constructed based on a total effect of 100bp estimated by Eser et al. (2019). This total effect is then divided into four parts (shocks) consistent with the weights of the Eurosystem asset purchase packages, see Table 6.4.1. The first package is treated in a specific way, as it was partially anticipated in 2014Q4 and detailed in 2015Q1. In this case we assume an ad hoc repartition of 1/3 and 2/3 between the quarters.

We set the total effect on the effective exchange rate to 9%. This number comes from a 12%-effect on the euro-dollar exchange rate estimated by Dedola et al. (2018) and the assumption that other European currencies (except the British pound) remain pegged to the euro. We then divide it into four shocks using the same method, see Table 6.4.1.

Table 6.4.1: Eurosystem asset purchase packages and their effects on the term premium and the nominal exchange rate

Package	Amount	Term prem. effect	Exch. rate effect
January 2015	1140	-21.8 in 2014Q4	-1.9 in 2014Q4
		-43.7 in 2015Q1	-3.8 in 2015Q1
December 2015	360	-20.7 in 2015Q4	-1.8 in 2015Q4
March 2016	240	-13.8 in 2016Q1	-1.2 in 2016Q1
Total	1740	-100	-9

Note: Package amounts measured in billions of euros. Effects of shocks are annualized basis points. Effects realized in 2014Q4 are assumed to be anticipatory for the package of January 2015.

[99] We would like to thank Stephane Dees for supplying the data on shocks, for clarifying their construction and for offering helpful comments.

[100] We omit explicit modeling of effects on stock prices, as FR-BDF accounts for such movements implicitly by modeling cost of equity via the long-run rate which itself is affected by the term premium.

Our main results under VAR-based expectations – presented in the first row of Table 6.4.2 and in Figure 6.4.1 – indicate that the APP had notable real and nominal effects on the French economy.[101] The table shows yearly inflation and GDP growth rates which are substantial and positive. Overall, APP boosts investment and exports through a fall in long interest rates and a depreciating exchange rate. On the nominal side, it first results in imported inflation and then in domestic inflation due to a rise in factor costs. The second and third rows of Table 6.4.2 show simulation results from experiments where it is assumed that the APP affect only the term premium and exchange rates – the movements in the nominal and real side of the economy mainly appear to be the result of movements in exchange rates.

In contrast, under MCE, we find that in terms of the GDP growth rate the difference in the short run is not very striking, although inflation reacts more strongly as expectations regarding positive economic developments in the future result in a more pronounced Phillips effect today. Effects on investment are accentuated but simultaneously, households expect to lose some purchasing power and thus do not raise their consumption as they would otherwise.

The importance of this inflationary effect is also demonstrated in the fifth and sixth rows of Table 6.4.2. First, under MCE the inflationary effect of exchange rate movements is much larger. Furthermore, note that under MCE, the share of the total real effects arising from the exchange rate channel is smaller than under VAR-based expectations, and that in the long run GDP growth turns negative. This is due to the faster adjustment of the economy in comparison to the VAR case. This leads to a reduction in purchasing power with negative consequences in the medium run on consumption. In addition, competitiveness reverts to its pre-shock level as a result of domestic inflation. Note that in the VAR case these effects will also eventually lead to negative GDP growth.

Finally, two additional experiments were conducted in the same framework. In the first we investigated the importance of the assumed dynamics of the persistence of the shock. In this robustness check we instead assume that the shock persists only for a single quarter and then revert with its historical persistence. We find that when agents form their future beliefs using the VAR framework, the differences – with respect to responses obtained under the baseline assumptions regarding the persistence of the shocks – are at the start of the simulation relatively minor, with e.g. the peak effect on year-on-year GDP growth being only 0.02pp larger. At the end of the simulation period the differences increase, so that the difference in GDP growth rates is roughly -0.15pp and -0.1pp for inflation. Under the forward looking MCE framework the results are qualitatively similar for GDP, but stronger. For inflation this difference in assumptions plays a rather large role: under MCE the peak effect is reduced by -0.25. At the end of simulation the difference in GDP growth rates is roughly -0.1; inflation is reduced by -0.3.

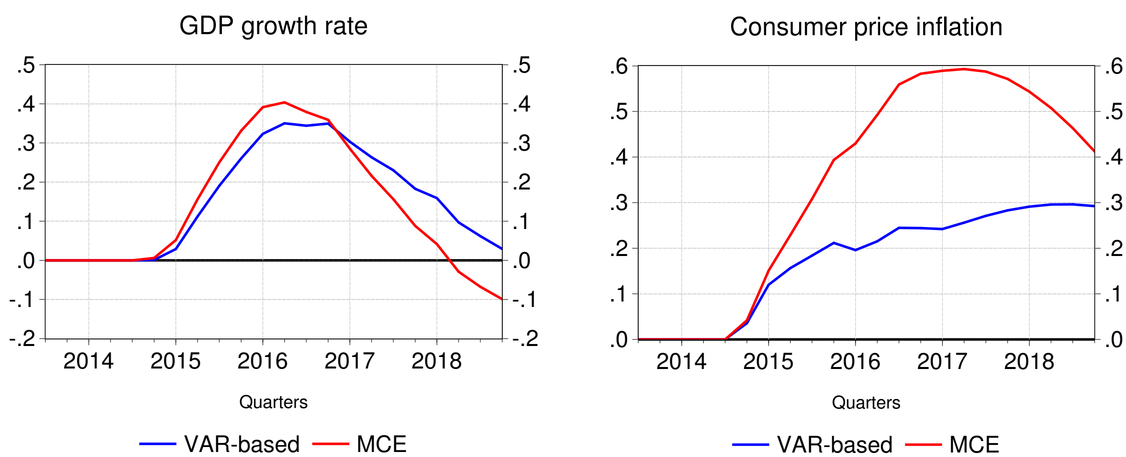
In the second experiment we studied the importance of Euro area dynamics. We carry out this exercise with the help of a satellite model, which is constructed using tables of impulse responses that result from the Eurosystem's Basic Model Elasticity (BME) exercise. The tables describe the responses of each individual country to a variety of economic shocks in the models of each national central bank participating in the exercise; based on the tables it is possible to conduct an analysis of the joint response of the euro area. We construct our

[101] Note that the slight fall in inflation at the start of 2016 shown in Figure 6.4.1 is explained by a basis effect arising from the year-on-year computation of the growth rate.

alternative experiment by applying the shocked deviations of real demand for French imports and the prices of foreign exports as additional innovations to the French economy.[102] Both of these variables exhibit a similar dynamic, where they at first increase and then revert strongly after 2016.

We find that for output the cumulated response of the growth rate over the period 2015-2018 remains almost unchanged. However, the response is now more front-loaded, with a greater response occurring in 2015 and 2016, and conversely a smaller response occurring in 2017 and 2018. In contrast to output, the response of inflation is unambiguously stronger, although not very much so, as the cumulated increase is just 0.06pp over the simulation period.

Figure 6.4.1: Year-on-year French output growth and inflation (percent), deviation from baseline



[102]The satellite model also produces euro area output and price level as outputs. Transformations of these variables appear in the expectations of FR-BDF. An alternative exercise was also conducted where the effect of these expectations shocks was taken into account. As the difference turned out to be very small, we omit the results of this second exercise. Further details are available on request.

Table 6.4.2: The effect of APP on average year-on-year inflation and GDP growth, percent

		GDP growth					Inflation				
		2015	2016	2017	2018	**Total**	2015	2016	2017	2018	**Total**
VAR	Full	0.15	0.34	0.24	0.09	**0.82**	0.17	0.25	0.31	0.35	**1.08**
	TP	0.03	0.06	0.07	0.05	**0.21**	0.00	0.02	0.04	0.06	**0.13**
	ER	0.12	0.28	0.18	0.04	**0.62**	0.17	0.22	0.26	0.29	**0.95**
MCE	Full	0.20	0.38	0.19	-0.04	**0.73**	0.27	0.52	0.59	0.48	**1.85**
	TP	0.08	0.14	0.10	0.02	**0.35**	0.04	0.12	0.16	0.12	**0.44**
	ER	0.12	0.24	0.08	-0.06	**0.38**	0.23	0.39	0.42	0.35	**1.40**

Note: "Full" describes results from the full experiment, "TP" results from an experiment with a shock just to the term premium and "ER" a similar experiment with a shock to just the exchange rates.

7 Conclusion

In this paper, we have described in relative detail the features and applications of the new semi-structural model of the Banque de France for the French economy, FR-BDF. We would like to highlight here some of the key novelties in FR-BDF, particularly in comparison to the old forecasting model, Mascotte.

As was initially intended for the project, FR-BDF has a rich set of financial channels; e.g. shocks to interest rates or other financial variables of the model have economically meaningful and realistic effects on the macroeconomy, both real and nominal. This is in particular clearly demonstrated by the various experiments we have carried out for assessing monetary transmission and is in stark contrast to Mascotte, in which monetary policy shocks have for example relatively modest effects on macroeconomic dynamics. On the other hand, it would seem that some of the issues related to monetary policy shocks present in textbook DSGEs – where the interdependence of interest rates and real activity is known to be very strong – are not present in FR-BDF. FR-BDF does not suffer from the forward guidance puzzle.

When analyzing the dynamics of FR-BDF, explicit expectations play a large role. The expectations model E-SAT describes a backward-looking method for achieving this role for expectations, although FR-BDF can also be operated under model-consistent expectations. The economic impact of expectations can be seen from e.g. the various figures describing dynamic contributions. We would also like to emphasize the importance of how expectations are modeled in FR-BDF: Eurosystem asset purchase programmes have for example stronger effects on inflation under MCE in comparison to backward-looking expectations.

Furthermore, the model has a well-defined long run in the sense that it has a balanced growth path toward which it converges in various simulations. This feature is completely absent from many semi-structural models used in forecasting.

However, further research and development is still required. Two key model features, which were outside the scope of the original project, are planned to be included in the model: connecting the model to the euro area and accounting in more detail for the interaction between the macroeconomy and the financial system.

Connecting the French economy to the euro area in FR-BDF is a two-fold process. The first step consists of accounting for various indirect trade channels through which shocks affecting the euro area would be passed on to France. The second requires modelling endogenous monetary policy – as it is, it is assumed that French economic developments have no effect on the short rate. Given France's rather large share in the euro area economy, this is clearly a strong approximation on which it would be easy to improve, leading to gains in the accuracy of the model, in particular policy analysis. This extension would be separate from the forecasting use of the model, where it is not applicable for reasons described in section 4.11; instead it would only be applied in policy analysis.

The central features regarding the interaction between the macroeconomy and financial markets that FR-BDF still lacks are the causal channel through which the financial sector is affected by macroeconomic developments and the amplification that could arise from a financial accelerator at least in times of financial stress. The former would require careful modelling of the dependence of e.g. house and stock prices on aggregate dynamics. The latter would require major improvements in the way in which credit and leverage dynamics are modeled in order to be able to relate them to the financial conditions faced by households and firms.

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A Appendix

A.1 Stability conditions, E-SAT model

In order to simplify the analysis, we assume here that the French output gap does not react to that of the euro area ($\delta_q = 0$).[A.1] Without this feedback, the analysis of the dynamic of the system can be decomposed recursively into three blocks: AR(1) of the targets, the French economy and the euro area.

As the first block does not raise any issues, we start by looking at the second block, i.e. French variables Q_t and $\pi_{Q,t}$. Since the interest rate does not react to domestic variables, the Taylor rule does not play any role for ensuring the stability of this block. The stability of this block will depend on the 2×2 sub-matrix $H_{q,\pi}$, related to the dynamics of the output gap and inflation:

$$H_{q,\pi} = \begin{bmatrix} \lambda_q & \sigma_q \\ \kappa_\pi & \lambda_\pi \end{bmatrix}$$

The system is stable, if and only if the two eigenvalues of $H_{q,\pi} - I$ are negative. In the absence of any feedback of macro variables on the interest rate, stability occurs when the determinant $H_{q,\pi} - I$ is positive and its trace is negative[A.2]. These two conditions are equivalent to:

$$\begin{aligned} (1 - \lambda_q)(1 - \lambda_\pi) - \kappa_\pi \sigma_q &> 0 \\ (1 - \lambda_q) + (1 - \lambda_\pi) &> 0 \end{aligned}$$

The first condition requires that the destabilizing role of the real interest rate (related to $\kappa_\pi \sigma_q$ and λ_q) be sufficiently small compared to the anchoring degree of inflation $(1 - \lambda_\pi)$ and of the output gap $(1 - \lambda_q)$. The second condition should be always verified, provided we have some anchoring of inflation and the output gap ($\lambda_\pi < 1$ and $\lambda_q < 1$).

Concerning the third block of the euro area, if there were no feedback from the Taylor rule ($\alpha_i = \beta_i = 0$), stability conditions would be the same for parameters $\lambda_{q,ea}$, λ_π, $\kappa_{\pi,ea}$ and $\sigma_{q,ea}$. With a feedback of monetary policy on macro variables, the stability should be reinforced and, hence, these stability conditions should be sufficient.

A.2 Implications for the term premium, E-SAT model

There are two types of bonds in FR-BDF: the classical one-period bond that returns R_t^S, and the long-term bond that returns R_t^L. The term premium is defined as the difference between the return on the long-term bond and the expected return on the short-term one, where the latter is the expected sum of all future short-term rates over a horizon equal to

[A.1] Our numerical simulations show that this channel does not change qualitatively the dynamics of E-SAT.

[A.2] This condition only ensures stability of the output gap and inflation. In order to achieve stability of the real exchange rate (here, p_{ea}/p_Q), we could have considered adding a competitiveness channel, i.e. adding this variable within the IS curve. However, (i) the identification of the parameter corresponding to this channel is very weak; (ii) it would create either persistent responses or oscillations depending on the size of this parameter.

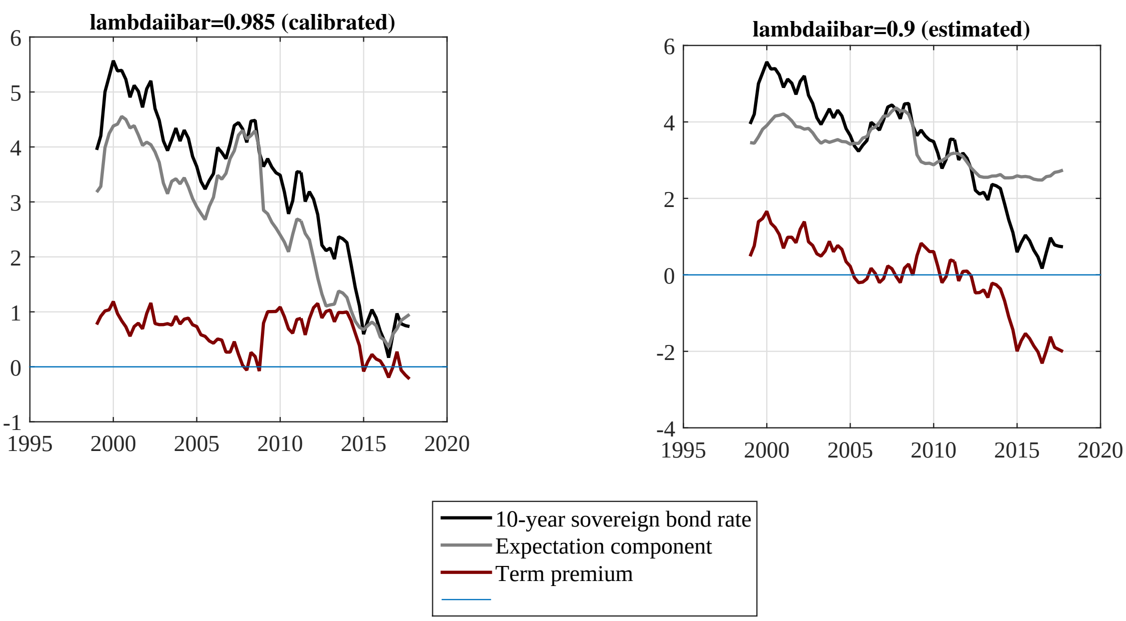
the maturity of the corresponding long-term asset:

$$R_t^L - (1-p^L)\sum_{z=0}^{\infty} p^L R_{t+z}^S.$$[A.3]

The term premium is unobserved and in order to compute it we approximate R_t^L with the return on the euro area 10 year government bond, R_t^S is the 3-month Euribor and $p^L = 1 - \frac{1-1/\hat{R^L}}{1-1/(\hat{R^L})^{40}}$, where $\hat{R^L} = (1+\bar{R_t^L}/100)$. The expected return of the short-term bond is computed by reversing the E-SAT model, i.e. $(1-p^L)\sum_{z=0}^{\infty} p^L R_{t+z}^S = (1-p^L)Z_t[(I-p^L H)^{-1}]'$, for the definitions of Z and H see equation 1.

We present the term premium consistent with our estimated VAR in the right-hand plot of Figure A.1. Starting from 2012, the term premium is negative, which contradicts the results of the term structure model of the Banque de France. In order to bring our model in line with the official views of the Banque de France, we calibrate λ_i parameter to 0.985, see left-hand plot of Figure A.1.

Figure A.1: 10-year bond yield, expected nominal interest rate and term premium.



[A.3] For detailed derivations see the note DEMFI_2016_032.

A.3 Additional figures

Figure A.1: Prior (gray) and posterior distributions (black).

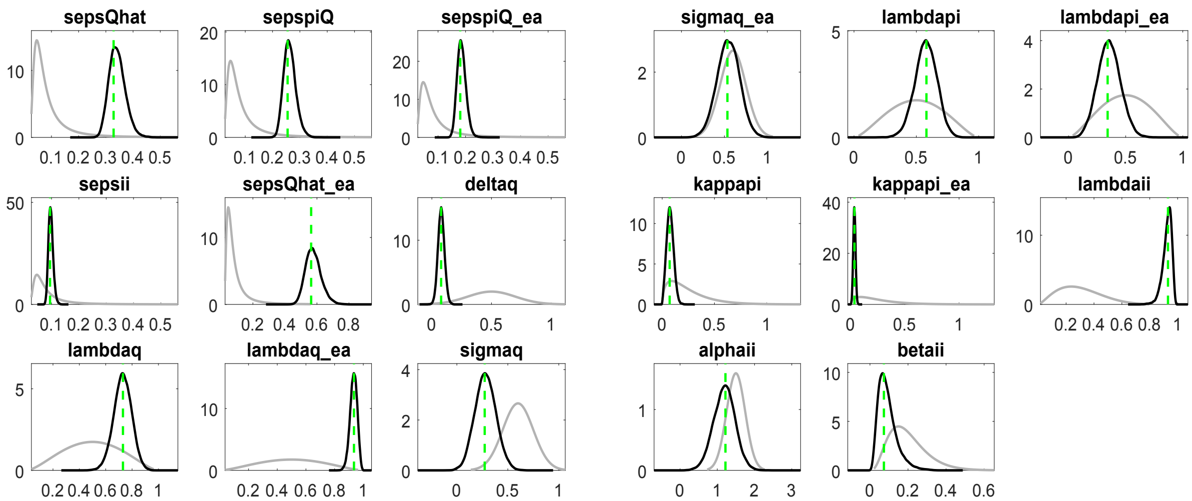


Figure A.2: GDP components' responses to a monetary policy shock, see section 6.2